10.5:

We can derive that
$$\frac{\partial S(q,\alpha,t)}{\partial t} = -\frac{1}{2}m\omega^2(\alpha^2+q^2-2\alpha q\cos\omega t)\csc^2\omega t$$
and
$$\frac{\partial S(q,\alpha,t)}{\partial q} = m\omega(q\cos\omega t - a)\csc\omega t.$$
It is obvious that we can write the Hamiltonian as,
$$H\left(q,\frac{\partial S}{\partial q},t\right) = \frac{1}{2}m\omega^2[(q\cos\omega t-a)^2\csc^2\omega t + q^2].$$
Therefore, the principal function satisfies,
$$\frac{\partial S(q,\alpha,t)}{\partial t} + H\left(q,\frac{\partial S}{\partial q},t\right) = 0,$$
which is a solution of the Hamiltonian-Jacobi. And clearly, by the Hamilton's equations,
$$\dot{p} = -\frac{\partial H}{\partial q} = -m\omega^2 q,$$
$$\dot{q} = \frac{\partial H}{\partial p} = \frac{p}{m}.$$
It leads that
$$\ddot{q}+\omega^2 q = 0.$$
Thus, this function generates the solution to the motion of the harmonic oscillator.

10.11:

We have
$$\dot{z} = -\frac{4\pi\dot{x}}{\lambda}A\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi x}{\lambda}.$$
Thus,
$$L = \frac{1}{2}m\dot{x}^2\left(1+\frac{16\pi^2A^2}{\lambda^2}\cos^2\frac{2\pi x}{\lambda}\sin^2\frac{2\pi x}{\lambda}\right) - mg\cos^2\frac{2\pi x}{\lambda}.$$
And as a result, we have
$$p = \frac{\partial L}{\partial \dot{x}} = m\dot{x}\left(1+\frac{16\pi^2A^2}{\lambda^2}\cos^2\frac{2\pi x}{\lambda}\sin^2\frac{2\pi x}{\lambda}\right).$$
And
$$H = p\dot{x} - L = \frac{p^2}{2m\left(1+\frac{16\pi^2A^2}{\lambda^2}\cos^2\frac{2\pi x}{\lambda}\sin^2\frac{2\pi x}{\lambda}\right)} + mgA\cos^2\frac{2\pi x}{\lambda}.$$
And naturally, we have
$$\frac{dx}{dt} = \frac{\partial H}{\partial p} = \frac{p}{m\left(1+\frac{4\pi^2A^2}{\lambda^2}\sin^2\frac{4\pi x}{\lambda}\right)}.$$

$$\frac{dp}{dt} = -\frac{\partial H}{\partial x} = -\frac{p^2}{m\left(1+\frac{4\pi^2 A^2}{\lambda^2}\sin^2\frac{4\pi x}{\lambda}\right)^2}\frac{4\pi^2 A^2 \sin\frac{4\pi x}{\lambda}\cos\frac{4\pi x}{\lambda}}{\lambda^2} - \frac{4\pi}{\lambda}mgA\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi x}{\lambda}.$$

This is the evolution of the system in the phase space.

10.14:

The Hamiltonian is

$$H\left(x, \frac{\partial S}{\partial x}\right) = \frac{1}{2m}\left(\frac{\partial S}{\partial x}\right) - \frac{k}{|x|}.$$

The Hamilton-Jacobi equation yields

$$\frac{1}{2m}\left(\frac{\partial S}{\partial x}\right) - \frac{k}{|x|} + \frac{\partial S}{\partial t} = 0.$$

Because the energy is conserved, we can take the principal function as the form

$$S = W(x, E) - Et + C.$$

Providing the energy is negative,

$$\frac{\partial W}{\partial x} = \sqrt{2m}\left[\frac{k}{|x|} - |E|\right]^{1/2}.$$

The turning point is at $x = \pm k/|E|$, and the action variable is,

$$J = \oint p dq = 4\sqrt{2m}\int_0^{k/|E|}\left[\frac{k}{x} - E\right]^{1/2} dx = 2\pi k\sqrt{\frac{2m}{|E|}}.$$

Then we can express the Hamiltonian as

$$H = E = -\frac{8m\pi^2 k^2}{J^2}.$$

The period is

$$T = 1/\nu = \left(\frac{\partial H}{\partial J}\right)^{-1} = \left(\frac{16m\pi^2 k^2}{J^3}\right)^{-1} = \frac{\pi k\sqrt{2m}}{|E|^{3/2}}.$$

10.17:

The Hamiltonian of projectile is given by

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + mgy.$$

The Hamiltonian-Jacobi equation reads

$$\frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2 + \frac{1}{2m}\left(\frac{\partial S}{\partial y}\right)^2 + mgy + \frac{\partial S}{\partial t} = 0.$$

Since the Hamiltonian does not depend on time explicitly, we can write the principal function as

$$S = -Et + W(x, y) + C.$$

Therefore, it turns out

$$\frac{1}{2m}\left(\frac{\partial W}{\partial x}\right)^2 + \frac{1}{2m}\left(\frac{\partial W}{\partial y}\right)^2 + mgy = E$$

Since the Hamiltonian can be decomposed as

$$H = f_1\left(x, \frac{\partial W}{\partial x}\right) + f_2\left(y, \frac{\partial W}{\partial y}\right),$$

we can also decompose W as

$$W = W_1(x) + W_2(y).$$

We have

$$\frac{1}{2m}\left(\frac{dW_1}{dx}\right)^2 = \alpha_1 \implies W_1 = \pm\sqrt{2\alpha_1 m}x,$$

$$\frac{1}{2m}\left(\frac{dW_2}{dy}\right)^2 + mgy = \alpha_2 \implies W_2 = \pm\frac{2\sqrt{2}(\alpha_2 - mgy)^2}{3g\sqrt{m(\alpha_2 - mgy)}},$$

$$\alpha_1 + \alpha_2 = E.$$

Therefore, the principal function is

$$S = -Et \pm \sqrt{2\alpha_1 m}x \pm \frac{2\sqrt{2}(\alpha_2 - mgy)^2}{3g\sqrt{m(\alpha_2 - mgy)}} + C.$$

We can derive

$$\beta_1 = \frac{\partial S}{\partial \alpha_1} = -t \pm \sqrt{\frac{m}{2\alpha_1}}x \implies x = \pm\sqrt{\frac{2\alpha_1}{m}}(\beta_1 + t),$$

$$\beta_2 = \frac{\partial S}{\partial \alpha_2} = -t \pm \frac{\sqrt{2m}}{mg}\sqrt{\alpha_2 - mgy} \implies y = \frac{\alpha_2}{mg} - \frac{g}{2}(\beta_2 + t)^2.$$

Putting initial conditions, we get

$$x = v_0 t\cos\alpha,$$

$$y = v_0 t\sin\alpha - \frac{1}{2}gt^2.$$

10.26:

The Lagrangian is

$$L = \frac{1}{2}I_1(\dot{\theta}^2 + \psi\sin^2\theta) + \frac{1}{2}I_3(\dot{\phi} + \psi\cos\theta)^2 - Mgl\cos\theta.$$

The conjugate momenta are

$$p_\theta = I_1\dot{\theta},$$

$$p_\psi = I_3(\dot{\psi} + \dot{\phi}\cos\theta),$$

$$p_\phi = I_3\dot{\psi}\cos\theta + \dot{\phi}(I_1\sin^2\theta + I_3\cos^2\theta).$$

After that, we have

$$E = \frac{1}{2} I_3 \omega_3^2 + \frac{I_1}{2} \frac{(b - a\cos\theta)^2}{\sin^2\theta} + Mgl\cos\theta.$$

Where

$$I_1 a = p_\phi,$$
$$I_2 b = p_\psi.$$

We assume that

$$\alpha = \frac{I_1}{2} \frac{(b - a\cos\theta)^2}{\sin^2\theta} + Mgl\cos\theta.$$

After separation of variables, we have

$$W = W(\theta) + W(\psi) + W(\phi).$$

Where

$$W(\psi) = I_1 a\psi,$$
$$W(\phi) = I_1 b\phi.$$

And

$$W(\theta) = \int d\theta \sqrt{2I_1\alpha - I_1^2 \frac{(b - a\cos\theta)^2}{\sin^2\theta} - 2I_1 Mgl\cos\theta}.$$

And

$$t + \beta_1 = \frac{\partial W}{\partial \alpha},$$
$$\beta_2 = \frac{\partial W}{\partial a},$$
$$\beta_3 = \frac{\partial W}{\partial b}.$$

And after rearranging parameters, we have

$$\dot{u}^2 = \left(1 - u^2\right)(\alpha - \beta u) - (b - au)^2.$$

Where

$$\alpha = \frac{2E - I_3\omega_3^2}{I_1},$$
$$\beta = \frac{2Mgl}{I_1}.$$