8.9:

It is straightforward to write down that,

$$S = \int_{t_1}^{t_2} \left(p_i dq_i - H dt - \lambda_k \psi_k dt\right).$$

Then we can define $H' = H + \lambda_k \psi_k$, the canonical equations become,

$$\begin{aligned} \dot{q}_i &= \frac{\partial H'}{\partial p_i} \\ \dot{p}_i &= -\frac{\partial H'}{\partial q_i} \end{aligned}.$$

That is,

$$\begin{aligned} \frac{\partial H}{\partial p_i} + \sum_k \lambda_k \frac{\partial \psi_k}{\partial p_i} &= \dot{q}_i \\ \frac{\partial H}{\partial q_i} + \sum_k \lambda_k \frac{\partial \psi_k}{\partial q_i} &= -\dot{p}_i \end{aligned}$$

Let's take a constraint as $\psi = H\left(q_1, \ldots, q_{n+1}; p_1, \ldots, p_n\right) + p_{n+1} = 0$, we can have

$$\begin{aligned} (1+\lambda)\frac{\partial H}{\partial p_i} &= \dot{q}_i \\ (1+\lambda)\frac{\partial H}{\partial q_i} &= -\dot{p}_i \\ \lambda = \lambda \frac{\partial \psi}{\partial p_{n+1}} &= \dot{q}_{n+1} = \frac{dt}{d\theta} \end{aligned}$$

8.16:

(a) By the Hamilton equation, we have

$$\dot{q} = \frac{\partial H}{\partial p} = \frac{p}{\alpha} - bqe^{-\alpha t},$$

$$\implies p = \alpha\left(\dot{q} + bqe^{-\alpha t}\right).$$

The Lagrangian is

$$L = p\dot{q} - H = L = \frac{\alpha}{2}\dot{q}^2 + \alpha bqe^{-\alpha t}\dot{q} - \frac{\alpha ab}{2}q^2 e^{-\alpha t} - \frac{kq^2}{2}.$$

(b) **(c)** It is easy to find that

$$L = \frac{\alpha}{2}\dot{q}^2 - \frac{k}{2}q^2 + \frac{d}{dt}\left(\frac{1}{2}abq^2 e^{-\alpha t}\right).$$

Therefore we can take

$$L' = \frac{\alpha}{2}\dot{q}^2 - \frac{k}{2}q^2,$$

and

$$H' = \frac{\alpha}{2}\dot{q}^2 + \frac{k}{2}q^2$$

The two Hamiltonians have the same physical meaning.

8.26:

(a) Take the Lagrangian as,

$$L = T - V = \frac{1}{2}m\dot{x}^2 - \frac{k_1}{2}x^2 - \frac{k_2}{2}(a-x)^2.$$

Therefore the Hamiltonian is,

$$H = T + V = \frac{1}{2}m\dot{x}^2 + \frac{k_1}{2}x^2 + \frac{k_2}{2}(a-x)^2.$$

There is no explicit time dependence of Hamiltonian. Thus the Hamiltonian and energy is conserved.

(b) Just substituting x with Q, we get,

$$L = \frac{1}{2}m(\dot{Q} + \omega b\cos\omega t)^2 - \frac{k_1}{2}(Q + b\sin\omega t)^2 - \frac{k_2}{2}(a - Q - b\sin\omega t)^2.$$

Now the canonical momentum is,

$$P = \frac{\partial L}{\partial \dot{Q}} = m(\dot{Q} + \omega b\cos\omega t).$$

Then the Hamiltonian is,

$$H = P\dot{Q} - L = \frac{1}{2}m\dot{Q}^2 - \frac{1}{2}\omega^2 b^2\cos^2\omega t + \frac{k_1}{2}(Q + b\sin\omega t)^2 + \frac{k_2}{2}(a - Q - b\sin\omega t)^2.$$

Now there is explicit time dependence of Hamiltonian, which is not conserved. However, the energy is conserved.

8.32:

Now we have

$$f(U) = \dot{U}^2 = \left(1 - U^2\right)(\alpha - \beta U) - (b - aU)^2$$

Where

$$U = \cos\theta$$

And θ is the nutation angle. Now we will set that

$$U = u_0 + u$$

Where u_0 is the steady state, and u is infinitesimal. Now we have

$$\beta u_0^3 - \left(\alpha + a^2\right) u_0^2 + (2ab - \beta) u_0 + \alpha - b^2 = 0$$

As well as

$$3\beta u_0^2 - 2\left(\alpha + a^2\right) u_0 + 2ab - \beta = 0$$

At this time, we have

$$\ddot{u} + \left(\alpha + a^2 - 3\beta u_0\right) u = 0$$

And notice that

$$\alpha = \frac{2Mgl\cos\theta_0}{I_1}$$
$$\beta = \frac{2Mgl}{I_1}$$
$$a = \frac{I_3}{I_1}\dot{\psi}_0$$
$$b = \frac{I_3}{I_1}\dot{\psi}_0 \cos\theta_0$$

So the top is nutating as a hormonic motion. That is,

$$\omega = \sqrt{\alpha + a^2 - 3\beta\cos\theta_0} = \sqrt{\frac{I_3^2}{I_1^2}{\psi_0}^2 - \frac{Mgl\cos\theta_0}{I_1}}.$$