QFT 讨论班

1、SR & Notation

$(t, x, y, z) \rightarrow$ 四维时空

$x^\mu = (x^0, x^1, x^2, x^3)$ Vector

时间 空间

$x_\mu = (x_0, x_1, x_2, x_3)$ Dual Vector

内积 Vector x Dual Vector $\rightarrow$ Number

$\boxed{1} + \boxed{1} = 2$ $\langle \psi | \psi \rangle$

Dual Vector 实

Dot product: $x^\mu x_\mu = A \cdot B = \sum a^1 b_1 + a_2 b_2 + a_3 b_3$

$= a^\mu b_\mu$

自由指标

SR. $s^2 = c^2 t^2 - x^2 - y^2 - z^2$

坐标变换不变

$x^\mu \cdot x_\mu = c^2 t^2 - x^2 - y^2 - z^2$

$$\begin{cases} x^\mu = (t, x, y, z) \\ x^\mu = (t, -x, -y, -z) \end{cases}$$

☐ 符号约定

1) $i\ j\ k \rightarrow$ 空间坐标指标

2) $\mu\ \nu \rightarrow$ 所有时空指标

3) 重复指标 $\rightarrow$ 求和

度规: x^μ x_μ $x^2 = g_{\mu\nu} x^\mu x^\nu = x^\mu x_\mu$

$$x^\mu = g^{\mu\nu} x_\nu \qquad x_\nu = g_{\mu\nu} x^\mu$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \qquad \eta_{\mu\nu} \to \text{Flat space}$$

微分算符

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = (\underset{\text{时间}}{\partial_0}, \partial_1, \partial_2, \partial_3) = (\frac{1}{c}\frac{\partial}{\partial t}, \nabla)$$

$c=1 \quad \hbar=1$

$$\partial^\mu = (\frac{\partial}{\partial t}, -\nabla)$$

d'Alembertian operator:

$$\Box = \partial^\mu \partial_\mu = \frac{\partial^2}{\partial t^2} - \nabla^2$$

$s^2 \to$ 相对论 $\qquad x$

$$E^2 - |p|^2 = m^2$$

$$\begin{cases} p^\mu = (\frac{E}{c}, \vec{p}) \\ p_\mu = (E, -\vec{p}) \end{cases}$$

相对论变换

$$\Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & -\gamma\beta & & \\ -\gamma\beta & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$x'^\nu = \Lambda^\nu{}_\mu x^\mu$$

$$x^a x_a = g_{ab} x^a x^b = g_{\mu\nu} \Lambda^\mu{}_a x^a \Lambda^\nu{}_b x^b$$

$$= g_{\mu\nu} x'^\mu x'^\nu$$

$x^\mu x_\mu =$

$$= x'^\mu x'_\mu = \text{Const}.$$

$$g_{\mu\nu} \Lambda^{\mu}{}_{a} \Lambda^{\nu}{}_{b} = g_{ab}$$
$$\uparrow \quad \uparrow$$

$$(\Lambda^{-1})^{\mu}{}_{\nu} \Lambda^{\rho}{}_{\nu} = \delta^{\mu}{}_{\nu} \Rightarrow (\Lambda^{-1})^{\mu}{}_{\rho} = g^{\mu\beta} g_{\alpha\rho} \Lambda^{\alpha}{}_{\beta}.$$

Lorentz 变换. $\Rightarrow$ group. (Lorentz group)

2.2. Klein - Gordon Equation.

For QM.

$$\underbrace{E^2 - p^2 = m^2}. \qquad E \to i\hbar\frac{\partial}{\partial t} \qquad p \to -i\hbar\nabla$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi + \frac{m^2c^2}{\hbar^2}\phi = 0$$

$$\underbrace{(\Box + m^2)\phi = 0} \qquad E = \pm(m^2c^4 + p^2c^2)^{\frac{1}{2}}$$

$$E = \frac{p^2}{2m} \Rightarrow \frac{\hbar^2}{2m}\nabla^2\phi = -i\hbar\frac{\partial\phi}{\partial t}.$$

$$\frac{d\rho}{dt} = 0 = \frac{\partial\rho}{\partial t} + \nabla\cdot\vec{j}$$

$$\begin{cases} \vec{j} = -\frac{i\hbar}{2m}(\phi^*\nabla\phi - \phi\nabla\phi^*). \\ \rho = \phi\phi^*. \end{cases}$$

For KG equation:

$$\begin{cases} \rho = \frac{i\hbar}{2m}\left(\phi^*\frac{\partial\phi}{\partial t} - \phi\frac{\partial\phi^*}{\partial t}\right) \\ \vec{j} = -\frac{i\hbar}{2m}\underbrace{(\phi^*\nabla\phi - \phi\nabla\phi^*)} \end{cases} \longrightarrow \phi = a + bi$$

$$\begin{cases} a = t \\ b = t \end{cases} \quad \rho < 0$$

$$j^{\mu} = (\rho, \vec{j})$$

$$A \overleftrightarrow{\partial_{\mu}} B = \frac{1}{2}[A \partial_{\mu} B - (\partial_{\mu} A) B]$$

$$j^{\mu} = \frac{i\hbar}{m} \phi^{*} \overleftrightarrow{\partial_{\mu}} \phi$$

$$\partial_{\mu} j^{\mu} = 0$$

$$\frac{i\hbar}{2m}(\phi^{*} \Box \phi - \phi \Box \phi^{*}) = 0$$