$$(\gamma^\mu p_\mu - m)\psi(p) = 0$$

$$p_0 = i\partial_0$$

$$(\gamma^0 p_0 - \gamma^i p_i - m)\psi(p) = 0$$

$$p_i = -i\partial_i$$

$$[i(\gamma^0\partial_0 + \gamma^i\partial_i) - m]\psi = 0$$

$$\gamma^0\partial_0 + \gamma^i\partial_i = \gamma^\mu\partial_\mu$$

$$\gamma^0\partial_0 - \gamma^i\partial_i \neq$$

$$\underline{(i\gamma^\mu\partial_\mu - m)\psi = 0}$$

$$i\gamma^\mu\partial_\mu$$

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix}$$

$$\underline{(\gamma^0)^\dagger = \gamma^0} \quad \gamma^{i\dagger} = -\gamma^i$$

$$(+\gamma^\mu\gamma^\nu\partial_\mu\partial_\nu + i\gamma^\mu\partial_\mu m)\psi = 0$$

$$\left[\frac{1}{2}\{\gamma^\mu, \gamma^\nu\}\partial_\mu\partial_\nu + m^2\right]\psi = 0$$

$$\gamma^\mu\gamma^\nu\partial_\mu\partial_\nu = \gamma^\nu\gamma^\mu\partial_\nu\partial_\nu$$

$$= \frac{1}{2}(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu)\partial_\mu\partial_\nu$$

$$= \frac{1}{2}\{\gamma^\mu, \gamma^\nu\}\partial_\mu\partial_\nu.$$

$$\downarrow$$

$$\frac{1}{2}\{\gamma^\mu, \gamma^\nu\}\partial_\mu\partial_\nu = \Box$$

$$(\Box + m^2)\psi = 0$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

$\mu = \nu = 0$

$$(\gamma^0)^2 = 1, \quad (\gamma^i)^2 = -1 \qquad \gamma^\mu\gamma^\nu = -\gamma^\nu\gamma^\mu$$

$$\gamma'^\mu = S\gamma^\mu S^{-1}, \quad S \text{ unitary}$$

$$\{\gamma'^\mu, \gamma'^\nu\}$$

$$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$= S\{\gamma^\mu, \gamma^\nu\}S^{-1} = 2S\, g^{\mu\nu} S^{-1}$$

$$= 2g^{\mu\nu} \qquad \psi' = S\psi.$$

$$(i\gamma^\mu\partial_\mu - m)\psi = 0$$

$$\psi^\dagger(+i\gamma^0\overleftarrow{\partial}_0 \mp i\gamma^i\overleftarrow{\partial}_i + m) = 0$$

$$\psi^\dagger\gamma^0(i\gamma^\mu\overleftarrow{\partial}_\mu + m) = 0$$

$\bar\psi = \psi^\dagger\gamma_0$ adjoint spinor

① $j^\mu = \bar\psi\gamma^\mu\psi$ $\boxed{\bar\psi\gamma^\mu\overleftarrow{\partial}_\mu}$

$$\partial_\mu j^\mu = \underline{(\partial_\mu\bar\psi)}\gamma^\mu\psi + \bar\psi\gamma^\mu\partial_\mu\psi$$

$$= im\bar\psi\psi + \bar\psi(-im\psi) = 0$$

$$j^0 = \bar\psi\gamma^0\psi = \psi^\dagger\psi = |\psi_0|^2 + |\psi_1|^2 + |\psi_2|^2 + |\psi_3|^2 > 0$$

② $(\gamma^\mu p_\mu - m)\psi = 0$

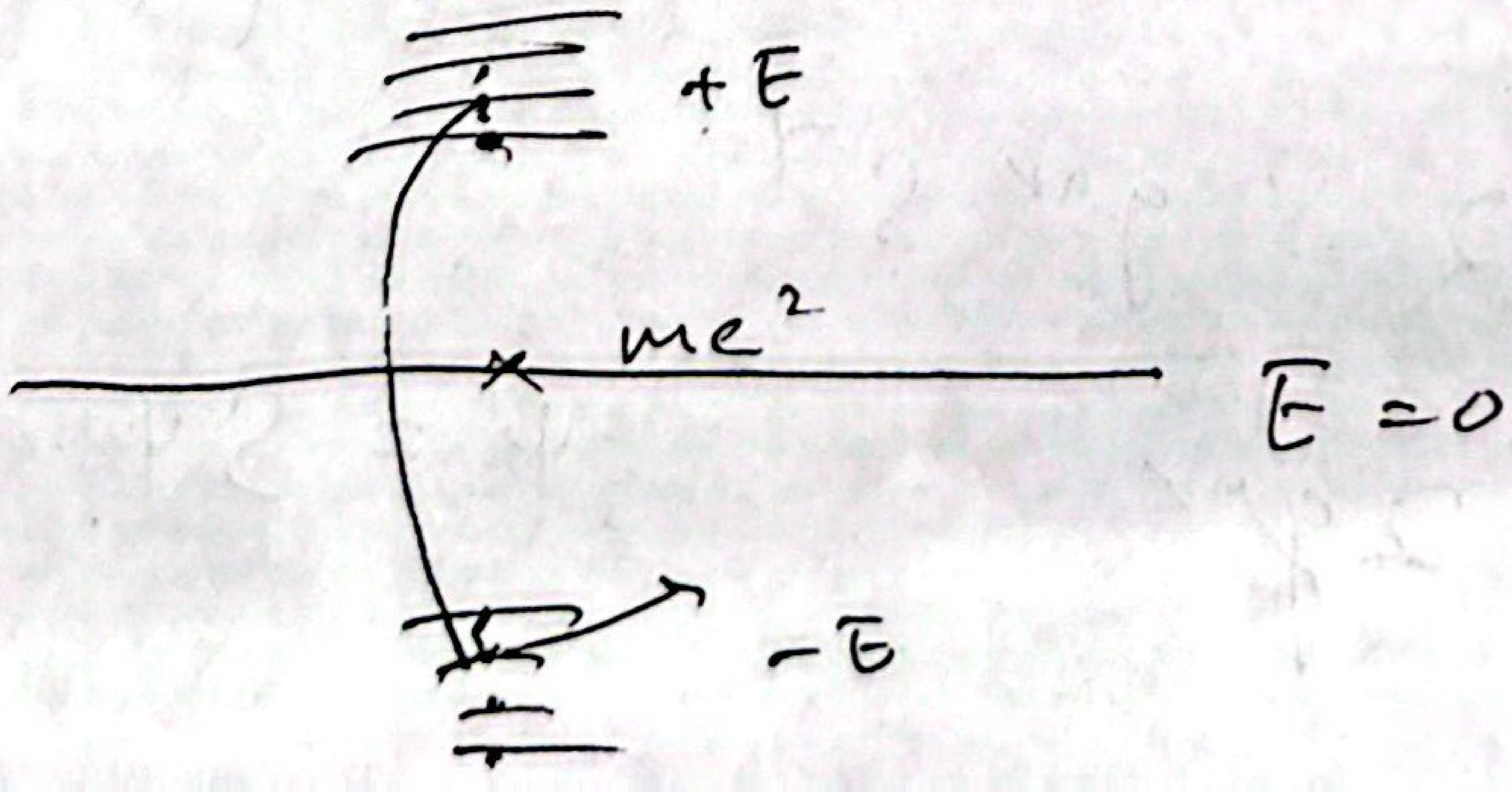
$\gamma^0 p_0\psi = m\psi$ $\quad \gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad$ 1, 1 / −1, −1

$$p_0\psi = \underline{m\gamma^0\psi} = \pm m\psi$$

$$p_0 = \pm m$$

$$p_0\psi = (m\gamma^0 - \gamma^0\gamma^i p_i)\psi$$

$$p_0 = \pm\sqrt{m^2 + p^2}$$



hole

E E 2E.

e^- + hole → energy

↓

positron

① $\psi = \begin{pmatrix} \phi_R \\ \phi_L \end{pmatrix}$ $\bar{\psi}\psi$

$$\phi_R' = \exp\left[\frac{i}{2}\vec{\sigma}(\vec{\theta} - i\vec{\phi})\right] \phi_R$$

$$\phi_L' = \exp\left[\frac{i}{2}\vec{\sigma}(\vec{\theta} + i\vec{\phi})\right] \phi_L .$$

$$\phi_R'^{\dagger} = \exp\left[-\frac{i}{2}\vec{\sigma}(\vec{\theta} + i\vec{\phi})\right] \phi_R^{\dagger}$$

$$\phi_L'^{\dagger} = \exp\left[-\frac{i}{2}\vec{\sigma}(\vec{\theta} - i\vec{\phi})\right] \phi_L^{\dagger}$$

$$\bar{\psi} = \psi^{\dagger}\gamma^0 = (\phi_R^{\dagger}, \phi_L^{\dagger})\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (\phi_L^{\dagger}, \phi_R^{\dagger}) \qquad \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

$$\bar{\psi}\psi = \underline{\phi_L^{\dagger}\phi_R} + \underline{\phi_R^{\dagger}\phi_L} \leftarrow \text{real - scalar.}$$

② $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\bar{\psi}\gamma^5\psi = (\phi_L^{\dagger}\ \phi_R^{\dagger})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \phi_R \\ \phi_L \end{pmatrix} = \phi_L^{\dagger}\phi_R - \phi_R^{\dagger}\phi_L$$

↪ pseudo - scalar

③ $\bar{\psi}\gamma^{\mu}\psi$

$$\bar{\psi}\gamma^0\psi = (\phi_L^{\dagger}\ \phi_R^{\dagger})\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \phi_R \\ \phi_L \end{pmatrix} = \phi_R^{\dagger}\phi_R + \phi_L^{\dagger}\phi_L$$

$$\bar{\psi}\vec{\gamma}\psi = \underline{\phi_R^{\dagger}\vec{\sigma}\phi_R} - \phi_L^{\dagger}\vec{\sigma}\phi_L$$

$$\begin{pmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$$

i) $\phi = 0$,

$\bar{\Psi} \vec{\gamma} \Psi$ $\qquad \theta \to 0.$

$$= \phi_R^\dagger \left(1 - \frac{i}{2}\vec{\sigma}\cdot\vec{\theta}\right) \vec{\sigma} \left(1 + \frac{i}{2}\vec{\sigma}\cdot\vec{\theta}\right) \phi_R .$$

$$= \phi_R^\dagger \left(\vec{\sigma} + \frac{i}{2} [\vec{\sigma}, \vec{\sigma}\cdot\vec{\theta}] \right) \phi_R .$$

$$= \phi_R^\dagger \left(\vec{\sigma} + \frac{i}{2} \underline{[\sigma_i, \sigma_j]} \theta_j \vec{e}_i \right) \phi_R .$$

$$= \phi_R^\dagger \left(\vec{\sigma} + (-1) \epsilon_{ijk} \sigma_k \theta_j \vec{e}_i \right) \phi_R .$$

$$= \phi_R^\dagger \left(\vec{\sigma} - \vec{\theta} \times \vec{\sigma} \right) \phi_R .$$

$$= \phi_R^\dagger \vec{\sigma} \phi_R - \vec{\theta} \times \left(\phi_R^\dagger \vec{\sigma} \phi_R \right) .$$

(ii) ———

(iii) Parity $\longrightarrow$ vector

④ $\bar{\Psi} \gamma^\mu \gamma^5 \Psi$ $\longrightarrow$ pseudovector

⑤ $\bar{\Psi} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) \Psi$

$$\begin{cases} \bar{\Psi}\Psi \\ \bar{\Psi}\gamma^5\Psi \\ \bar{\Psi}\gamma^\mu\Psi \\ \bar{\Psi}\gamma^\mu\gamma^5\Psi \\ \bar{\Psi}\underline{(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)}\Psi \end{cases}$$

$6 + 4 + 4 + 1 + 1 = 16$

Clifford Algebra

$\bar{\Psi} \Gamma \Psi$

$\begin{pmatrix} \phi_R \\ \phi_L \end{pmatrix}$ chiral rep $\quad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\frac{1 \pm \gamma^5}{2}$$

$$\frac{1+\gamma^5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad \frac{1-\gamma^5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\underline{\psi(x) = u(0)\, e^{-imt}} \qquad \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\underline{\psi(x) = v(0)\, e^{+imt}}$$

$$u^{(1)}(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad u^{(2)}(0) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad v^{(1)}(0) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad v^{(2)}(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Standard rep.

$$\gamma^0 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} = \begin{pmatrix} \mathbb{1} & \\ & -\mathbb{1} \end{pmatrix}$$

$$\gamma^0_{SR} = S\, \gamma^0_{CR}\, S^{-1}$$

$$\Rightarrow S = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \longrightarrow \psi = S \begin{pmatrix} \phi_R \\ \phi_L \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_R + \phi_L \\ \phi_R - \phi_L \end{pmatrix}$$

$$\psi' = M \psi$$

$$S\psi' = \boxed{S M S^{-1}}\, S\psi$$

$$\begin{pmatrix} e^{1/2\, \sigma\cdot\phi} & \\ & e^{-1/2\, \sigma\cdot\phi} \end{pmatrix} \longrightarrow \begin{pmatrix} \cosh\phi/2 & \vec{\sigma}\cdot\vec{n}\, \sinh\phi/2 \\ \vec{\sigma}\cdot n\, \sinh\phi/2 & \cosh\phi/2 \end{pmatrix}.$$

$$\tanh\phi/2 = \frac{p}{E+m}$$

$$\cosh\phi/2 = \sqrt{\frac{E+m}{2m}} \qquad \sinh\phi/2 = \sqrt{\frac{E-m}{2m}}$$

$M_{SR}(\vec{p}, E, m)$.

$$px = wt - \vec{p}\cdot\vec{x}$$

$$\psi^{(\alpha)}(x) = u^{(\alpha)}(p)\, e^{-ip\cdot x}$$

$$\psi^{(\alpha)}(x) = v^{(\alpha)}(p)\, e^{ipx}.$$

$$u^{(1)} = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} 1 \\ 0 \\ p_z/(E+m) \\ p_+/(E+m) \end{pmatrix} \qquad u^{(2)} = \alpha \begin{pmatrix} 0 \\ 1 \\ p_-/E+m \\ -p_z/E+m \end{pmatrix}$$

$$\alpha = \sqrt{\frac{E+m}{2m}}, \quad p_\pm = p_x \pm i p_y$$

$$v^{(1)} = \alpha \begin{pmatrix} p_z/E+m \\ p_+/E+m \\ 1 \\ 0 \end{pmatrix} \qquad v^{(2)} = \alpha \begin{pmatrix} p_-/E+m \\ -p_z/E+m \\ 0 \\ 1 \end{pmatrix}$$

$$\bar{u}^{(\alpha)}(p)\, u^{(\alpha)}(p) = 1.$$

$$= \left(\frac{E+m}{2m}\right)\left(1 \quad 0 \quad \frac{p_z}{E+m} \quad \frac{p_-}{E+m}\right) \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ p_z/E+m \\ p_+/E+m \end{pmatrix}$$

$$= \frac{E+m}{2m}\left(1 - \frac{p_z^2 + p_x^2 + p_y^2}{(E+m)^2}\right).$$

$$= \frac{E+m}{2m}\,\frac{2m}{E+m} = 1$$

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

$$\begin{cases} \bar{u}^{(\alpha)}(p)\, u^{(\alpha')}(p) = \delta_{\alpha\alpha'} \\ \bar{v}^{(\alpha)}(p)\, v^{(\alpha')}(p) = -\delta_{\alpha\alpha'} \\ \bar{u}^{(\alpha)}(p)\, v^{(\alpha')}(p) = 0. \end{cases}$$

$$(\gamma^\mu p_\mu - m)\, u(p) = 0$$

$$(\gamma^\mu p_\mu + m)\, v(p) = 0$$

$$\underline{(\not{p} - m)\, u(p) = 0}$$

$$(\not{p} + m)\, v(p) = 0$$

$$\bar{u}(p)(\not{p}-m)=0$$

$$\bar{v}(p)(\not{p}+m)=0.$$

$$\rho=\sum_i|\alpha_i\rangle\langle\alpha_i|$$

Def.

$$P_+=\sum_\alpha u^{(\alpha)}(p)\,\bar{u}^{(\alpha)}(p)$$

$$P_+^2= u^{(\alpha)}(p)\,\underline{\underline{\bar{u}^{(\alpha)}(p)\;u^{(\beta)}(p)}}\,\bar{u}^{(\beta)}(p)$$

$$= u^{(\alpha)}(p)\,\delta_{\alpha\beta}\,\bar{u}^{(\beta)}(p)$$

$$= u^{(\alpha)}(p)\,\bar{u}^{(\alpha)}(p) = P_+$$

$$(\not{p}-m)\,u(p)\bar{u}(p)=0$$

$$(\not{p}-m)\,P_+=0$$

$$\frac{\not{p}}{m}P_+=P_+=a+b\not{p}$$

$$\not{p}\,(a+b\not{p})=m(a+b\not{p})$$

$$\not{p}$$

$$\not{p}^2$$
$$=\gamma^\mu p_\mu\gamma^\nu p_\nu$$
$$=\gamma^\mu\gamma^\nu p_\mu p_\nu$$
$$=\frac{1}{2}\{\gamma^\mu,\gamma^\nu\}p_\mu p_\nu$$
$$=g^{\mu\nu}p_\mu p_\nu$$
$$=p\cdot p=m^2$$

$$a\not{p}+b\not{p}^2=ma+mb\not{p}\quad\Rightarrow\quad a=mb$$

$$bm^2=ma\quad\Rightarrow\quad a=mb$$

$$P_+^2=b^2(m+\not{p})(m+\not{p})=b(m+\not{p})$$

$$2m\not{p}\,b^2=b\not{p}\quad\Rightarrow\quad b=\frac{1}{2m}.$$

$$P_+ = \frac{\not{p}+m}{2m} \,,\quad P_- = \frac{-\not{p}+m}{2m}$$ 8

$$P_+ + P_- = 1$$

$$\boxed{\begin{aligned} \sum_\alpha u^{(\alpha)}(p)\,\bar{u}^{(\alpha)}(p) &= \frac{\not{p}+m}{2m} \\ \sum_\alpha v^{(\alpha)}(p)\,\bar{v}^{(\alpha)}(p) &= \frac{-\not{p}+m}{2m} \end{aligned}}$$

$$\vec{\mu} = \frac{e}{2m}\vec{l}$$

$$\vec{\mu}_s = \frac{e}{2m}\vec{s}\, g_s = \frac{e}{m}\vec{s} = \frac{e}{2m}\vec{\sigma}$$

$$p^\mu \longrightarrow p^\mu - eA^\mu$$

$$\gamma^0(E - e\phi)\psi - \vec{\gamma}\cdot(\vec{p} - e\vec{A})\psi = m\psi$$

$$\gamma^0 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

$$(E - e\phi)\begin{pmatrix} u \\ -v \end{pmatrix} - (\vec{p} - e\vec{A})\begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = m\begin{pmatrix} u \\ v \end{pmatrix}$$

$$\Rightarrow \begin{cases} (E - e\phi)u - \vec{\sigma}\cdot(\vec{p} - e\vec{A})v = mu \\ -(E - e\phi)v + \vec{\sigma}\cdot(\vec{p} - e\vec{A})u = mv \end{cases}$$

$$v = (E + m - e\phi)^{-1}\,\vec{\sigma}\cdot(\vec{p} - e\vec{A})u$$

$$= \underline{\frac{1}{2m}\vec{\sigma}\cdot(\vec{p} - e\vec{A})u} \propto O\left(\frac{v}{c}\right)u$$

$$Eu = \frac{\vec{\sigma}\cdot(\vec{p}-e\vec{A})\,\vec{\sigma}(\vec{p}-e\vec{A})}{2m}u + mu + e\phi u$$ 9.8.

$$(E-m)u = \left\{ \frac{[\vec{\sigma}\cdot(\vec{p}-e\vec{A})][\vec{\sigma}\cdot(\vec{p}-e\vec{A})]}{2m} + e\phi \right\} u.$$

Completeness relation.

$$[\vec{\sigma}\cdot(\vec{p}-e\vec{A})]^2$$

$$= (\vec{p}-e\vec{A})^2 + i\sigma(\vec{p}-e\vec{A})\times(\vec{p}-e\vec{A})$$

$$\underline{\hat{\vec{p}}\times\hat{\vec{A}} + \hat{\vec{A}}\times\hat{\vec{p}} \neq 0}.$$

$$[P_i, A_j] = -i\hbar\,\partial_i A_j$$

$$(P_iA_j - P_jA_i) + (A_iP_j - A_jP_i)$$

$$= -i\hbar(\partial_i A_j - \partial_j A_i)$$

$$\vec{p}\times\vec{A} + \vec{A}\times\vec{p}$$

$$= -i\hbar\,\nabla\times\vec{A} = -i\hbar\vec{B}$$

$$(E-m)u$$

$$= \frac{1}{2m}(\vec{p}-e\vec{A})^2 + e\phi$$

$$- \boxed{\frac{e\hbar}{2m}\vec{\sigma}\cdot\vec{B}}$$

$$\mu = \frac{e\hbar}{2m}\vec{\sigma}$$

$$= \frac{e}{2m}\vec{S}\cdot g_s.$$

$$g_s = 2$$