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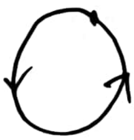
1. AB effect: $\mathcal{L} = (\partial_\mu\phi + ieA_\mu\phi)(\partial^\mu\phi - ieA^\mu\phi) - m^2\phi^*\phi - \boxed{\frac{1}{4}F^{\mu\nu}F_{\mu\nu}}$

$D_\mu = \partial_\mu + ieA_\mu$ $\qquad \vec{P} \to \vec{P} - e\vec{A}$

$\psi = |\psi|\exp(i\vec{p}\cdot\vec{r})$ $\qquad \psi \to \psi\cdot\exp[-ie\int\vec{A}\cdot d\vec{r}]$



loop: $\Delta\delta = e\oint\vec{A}\cdot d\vec{r} = e\int\nabla\times\vec{A}\cdot d\vec{s} = e\int\vec{B}\cdot d\vec{s} = e\Phi$

Gauge invariant

2. Berry phase: adiabatic process

$|\psi(0)\rangle = |n_0\rangle$ $\qquad H(t)$

$$|\psi(t)\rangle = \sum_m C_m(t)\cdot\exp\left[-i\int_0^t E_m(t')dt'\right]|m(t)\rangle$$

$$\sum_m\left[\dot{C}_m(t)|m(t)\rangle + C_m(t)|\dot{m}(t)\rangle\right] = 0$$

Notice: $\langle n(t)|\dot{m}(t)\rangle \approx 0$ $(n\neq m)$ $\qquad$ As $H(t)|m(t)\rangle = E_m(t)|m(t)\rangle$

$[H(t) - E_m(t)]|\dot{m}(t)\rangle = [\dot{E}_m(t) - \dot{H}(t)]|m(t)\rangle$ $\qquad \langle n(t)|\dot{m}(t)\rangle = \dfrac{\langle n(t)|\dot{H}(t)|m(t)\rangle}{E_m(t) - E_n(t)}$

$$\left|\frac{\langle n(t)|\dot{H}(t)|m(t)\rangle}{E_m(t) - E_n(t)}\right| \ll \langle m(t)|\dot{m}(t)\rangle$$

$$C_n(t) = \exp\left[-\int\langle n(t)|\dot{n}(t')\rangle dt'\right]$$

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$$\gamma_n = i\int_0^t \langle n(t)|\dot{n}(t')\rangle\, dt', \quad e^{i\gamma_n}$$

In parameter space(R): rewrite γ_n

$$\gamma_n = i\int_{R(0)}^{R(t)} \langle n(R)|\nabla_R n(R)\rangle\, dR \qquad C_n(t) = e^{i\gamma_n}$$

γ_n: gauge - dependent $\begin{cases} |n(R)\rangle \Rightarrow e^{i\xi(R)}|n(R)\rangle \\ \gamma_n(R) \Rightarrow \gamma_n(R) - \frac{\partial}{\partial R}\xi(R) \end{cases}$

demand: $\xi(R(0)) - \xi(R(T)) = 2\pi\cdot k \quad k\in\mathbb{Z}$

3. Berry curvature

$\vec{A}^n = \langle n(R)|\nabla_R n(R)\rangle$: Berry connection

$$\Omega^n_{\mu\nu}(R) = \frac{\partial}{\partial R^\mu}A^n_\nu(R) - \frac{\partial}{\partial R_\nu}A^n_\mu(R) = i\left[\left\langle \frac{\partial n(R)}{\partial R^\mu}\middle|\frac{\partial n(R)}{\partial R^\nu}\right\rangle - \left\langle \frac{\partial n(R)}{\partial R^\nu}\middle|\frac{\partial n(R)}{\partial R^\mu}\right\rangle\right.$$

If we set $\gamma_n = i\oint \langle n(R)|\nabla_R n(R)\rangle dR$, then

$$\gamma_n = \int_S dR^\mu \wedge dR^\nu \cdot \frac{1}{2}\Omega^n_{\mu\nu}(R)$$

3- dimension: $\vec{\Omega}_n(\vec{R}) = \nabla_R \times \vec{A}^n$

using $\langle n|\frac{\partial H}{\partial R}|n'\rangle = \langle\frac{\partial n}{\partial R}|n'\rangle(E_n - E_{n'})$

$$\Omega^n_{\mu\nu}(\vec{R}) = i\sum_{n'\neq n}\frac{\langle n|\frac{\partial H}{\partial R^\mu}|n'\rangle\langle n'|\frac{\partial H}{\partial R^\nu}|n\rangle - \langle n|\frac{\partial H}{\partial R^\nu}|n'\rangle\langle n'|\frac{\partial H}{\partial R^\mu}|n\rangle}{(E_n - E_{n'})^2}$$

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Notice: if there's degeneracy: non-Abelian gauge

$U(N)$ gauge field (Shindou and Imura, 2005)

We will focus on Abelian theory

$\gamma_n = \int_S dR^{\mu} \wedge dR^{\nu} \frac{1}{2} \Omega^n_{\mu\nu}$: integrate over Brillouin zone

If $\vec{A}^n(\vec{R}) = \langle n(R) | \nabla_R n(R) \rangle = \nabla \phi$, $\gamma_n = 0$

You can annihilate it through gauge transformation $\vec{A}'^n = \vec{A}^n - \nabla\phi = 0$

$|n(\vec{R})\rangle \to e^{i\phi(\vec{R})} |n(\vec{R})\rangle$ *

But for non-trivial topology, it's impossible.

4. Chern number

We can divide the zone into different parts with local gauge

focus on edge

the behavior on edge is singular

assume that on edge, $\vec{A}_2(\vec{R}) - \vec{A}_1(\vec{R}) = \Delta\Lambda$

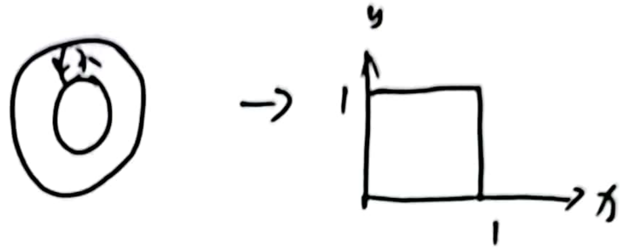
Given (*), $\Lambda = 2\pi \cdot n$ $(n \in \mathbb{Z})$

$\therefore \gamma_n = \sum_i \oint_{\partial \Sigma_i} \Delta\Lambda = 2\pi n,\ n \in \mathbb{Z}$ $C = \frac{\gamma_n}{2\pi}$ chern number

this is winding number

Usage: band structure

example: Brillouin zone



$$C=\frac{1}{2\pi}\left[\theta_x(1)-\theta_x(0)+\theta_y(0)-\theta_y(1)\right]$$

why it's an integer?

Gauss-Bonnet theorem

$$\int_M K \, dvol = 2\pi\chi(M)$$

K: 两个主曲率的积

$\chi(M) = V - E + F$

V: 顶点, E: 边, F: 面

for turos, it's 0 (it has a hole)

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Y-M Field: $\begin{cases} \text{QCD: } SU(3) \\ \text{电弱: } SU(2) \end{cases}$

$\delta\phi = -\Lambda \times \phi$

$$\left.\begin{aligned} D_\mu\phi &= \partial_\mu\phi + g\,\vec{W}_\mu \times \vec{\phi} \\ \delta(D_\mu\phi) &= -\Lambda \times (D_\mu\phi) \end{aligned}\right\} \Rightarrow W_\mu \to W_\mu - \Lambda \times W_\mu + \frac{1}{g}\partial_\mu\Lambda$$

$$\mathcal{L} = \frac{1}{2}(D_\mu\phi)(D^\mu\phi) - \frac{m^2}{2}\phi\cdot\phi - \frac{1}{4}W_{\mu\nu}\cdot W^{\mu\nu}$$