ϕ^4

we have prove that

$$Z[J] = N \exp\left[i \int \mathcal{L}_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right) dx\right] Z_0[J]$$

$$\mathcal{L}_{int} = -\frac{g}{4!}\phi^4$$

$$Z[J] = \frac{\exp\left[i\int L_{int}\left(\frac{1}{i}\frac{\delta}{\delta J}\right)dz\right] \exp\left[-\frac{i}{2}\int J(x)\,\Delta_F(x-y)\,J(y)\,dx\,dy\right]}{\left[\exp \;-\; \text{———} \;-\; \text{———}\right]_{J=0}}$$

$$\left[1 - \frac{ig}{4!}\int\left(\frac{1}{i}\frac{\delta}{\delta J}\right)^4 dz + \dots\right] \exp\left[-\frac{i}{2}\int J(x)\,\Delta_F(x-y)\,J(y)\,dx\,dy\right]$$

To order 0, free $Z_0[J]$,

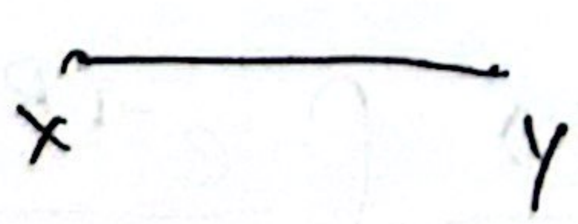
to order g,

$$\frac{1}{i}\frac{\delta}{\delta J(z)}\exp[\quad] = -\int \Delta_F(z-x)J(x)dx \, \exp(\quad)$$

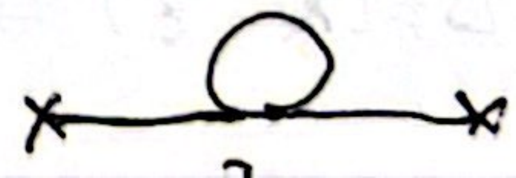
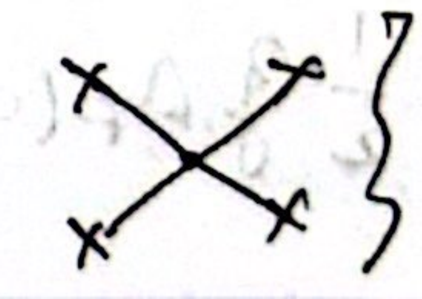
$$\left(\frac{1}{i}\frac{\delta}{\delta J}\right)^2 \exp[\quad] = \left\{i\Delta_F(0) + \left[\int \Delta_F(z-x)J(x)dx\right]^2\right\} \exp \,-$$

$$\left(\frac{1}{i}\frac{\delta}{\delta J}\right)^3 \exp(\quad] = \Big\{3[-i\Delta_F(0)]\int \Delta_F(z-x)J(x)dx$$
$$-\left[\int \Delta_F(z-x)J(x)dx\right]^3\Big\} \times \exp.$$

$$\left(\frac{1}{i}\frac{\delta}{\delta J}\right)^4 \exp[\,] = \Big\{-3[\Delta_F(0)]^2 + 6i\Delta_F(0)\left[\int\Delta_F(z-x)J(x)dx\right]^2$$
$$+\left[\int \Delta_F(z-x)J(x)dx\right]^4\Big\} \times \exp.$$

$\Delta_F(x-y)$ 

$\Delta_F(0)$ 

$\{-3$  $+ 6i$  $+$  $\} \times \exp$

exp 分母 $= 1 - \frac{ig}{4!}\int (-3\infty)\, dz$

$$Z[J] = \frac{\left[1 - \frac{ig}{4!}\int \left(-3\infty + 6i \quad + \quad dz\right]\right] \times \exp}{1 - \frac{ig}{4!}\int -3\infty\, dz}$$

$$= \left[1 - \frac{ig}{4!}\int \left(6i \quad + \quad \right) dz\right] \times \exp.$$

∞ disappear !

---

2 - point func.

$$\tau(x_1, x_2) = -\left.\frac{\delta^2 Z[J]}{\delta J(x_1)\,\delta J(x_2)}\right|_{J=0}$$

$Z[J] \rightsquigarrow 1 \longrightarrow i\Delta_F(x_1 - x_2)$

$\longrightarrow$ No

$$\frac{g}{4}\Delta_F(0)\int dx\, dy\, dz\; \Delta_F(z-x)\, J(x)\; \Delta(z-y)\, J(y)\; \exp(\quad)$$

$$= -\frac{g}{2}\Delta_F(0)\int dz\; \Delta_F(z - x_1)\,\Delta_F(z - x_2)$$

$\tau(x_1, x_2) = i \quad - \frac{g}{2} \quad + \cdots$

$$i\frac{1}{(2\pi)^4}\int \frac{e^{-ik(x-y)}}{k^2 - m^2 + i\epsilon}\, d^4k$$

$\longrightarrow$ renormalizatio

$$-\frac{1}{2}g\Delta_F(0)\int \Delta_F(x_1 - z)\,\Delta_F(x_2 - z)\,dz = -\frac{g}{2}\frac{\Delta_F(0)}{(2\pi)^8}\int \frac{e^{-ip(x_1-z)}}{p^2 - m^2 + i\epsilon}\frac{e^{-iq(x_2-z)}}{q^2 - m^2 + i\epsilon}\, dp\, dq\, dz$$

$$\cancel{\tau(x_1,x_2)} = \frac{i}{\cancel{(2\pi)^4}} \quad = -\frac{g}{2}\frac{\Delta_F(0)}{(2\pi)^4}\int \frac{e^{-ip(x_1-x_2)}}{(p^2-m^2+i\epsilon)^2}\,dp\,dq\,\delta^4(p+q)$$

$$= -\frac{g}{2}\frac{\Delta_F(0)}{(2\pi)^4}\int \frac{e^{-ip(x_1-x_2)}}{(p^2-m^2+i\epsilon)^2}\,dp.$$

$$\tau(x_1,x_2) = \frac{i}{(2\pi)^4}\int \frac{e^{-ip\cdot(x_1-x_2)}}{p^2-m^2+i\epsilon}\left[1+\frac{\frac{i}{2}g\,\Delta_F(0)}{p^2-m^2+i\epsilon}\right]d^4p.$$

$$= \frac{i}{(2\pi)^4}\int \frac{e^{-ip(x_1-x_2)}}{p^2-m^2-\frac{1}{2}ig\,\Delta_F(0)+i\epsilon}\,dp.$$

$m^2+\frac{1}{2}ig\,\Delta_F(0) = m_r^2.$ physical mass

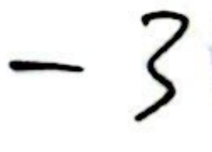
$$\delta m^2 = \frac{1}{2}ig\,\Delta_F(0).$$

↑

divergent !

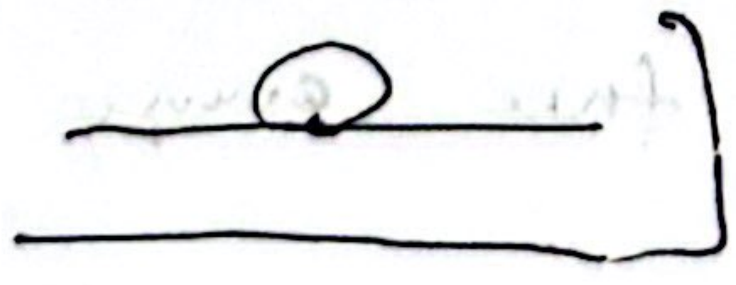
4 - point .

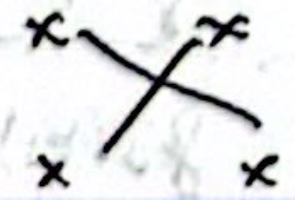
$$\tau(x_1,x_2,x_3,x_4) = \left.\frac{\delta^4 Z[J]}{\delta J(x_1)\,\delta J(x_2)\,\delta J(x_3)\,\delta J(x_4)}\right|_{J=0}$$

①  $= -3($  $) + \cdots$

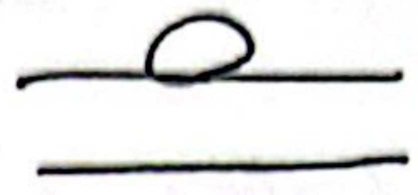
$$\frac{g}{4}\frac{\delta^4}{\delta J(x_1)\delta J(x_2)\,\delta J(x_3)\delta J(x_4)}\left\{\Delta_F(0)\int dx\,dy\,dz\,\Delta_F(x-z)\Delta_F(y-z)J(y)J(x)\ \exp\right\}\Big|_{J=0}$$

$$= \frac{-ig}{2}\Delta_F(0)\int dz\left[\Delta_F(z-x_1)\Delta_F(z-x_2)\Delta_F(x_3-x_4) + \cdots \#6\right]$$

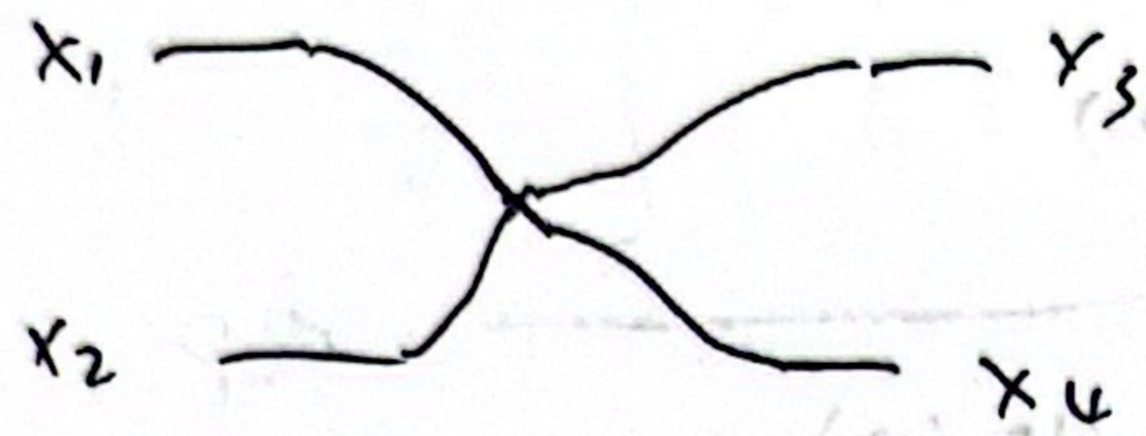
$= -3ig\,[$  $]$

②  $\quad \frac{-ig}{4!}\frac{\delta^4}{\ }\left\{ X \exp\right\}\Big|_{J=0} = -ig\,X$

$\tau(x_1,x_2,x_3,x_4)$

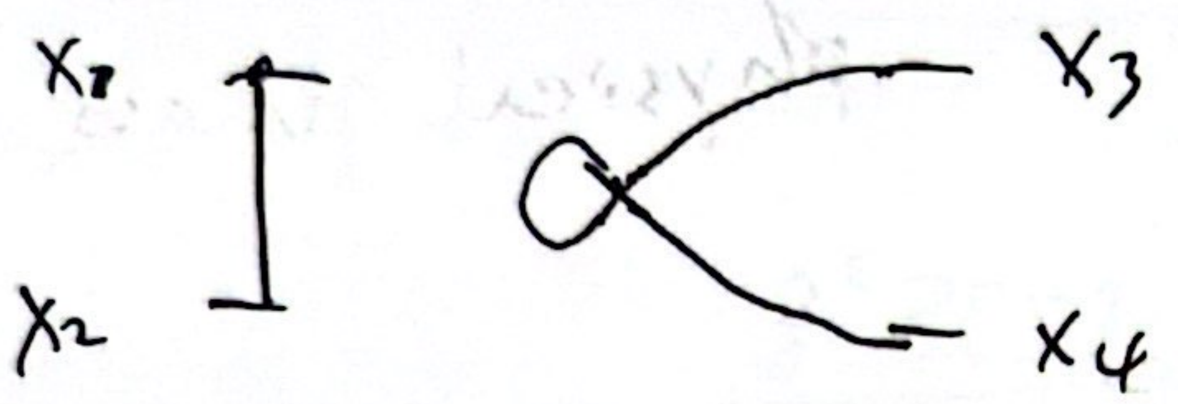
$$= -3[=] - \frac{ig}{4!}\left[12\times 6\left(\right) + 24(\times)\right]$$


phediagram g' $\times$

①


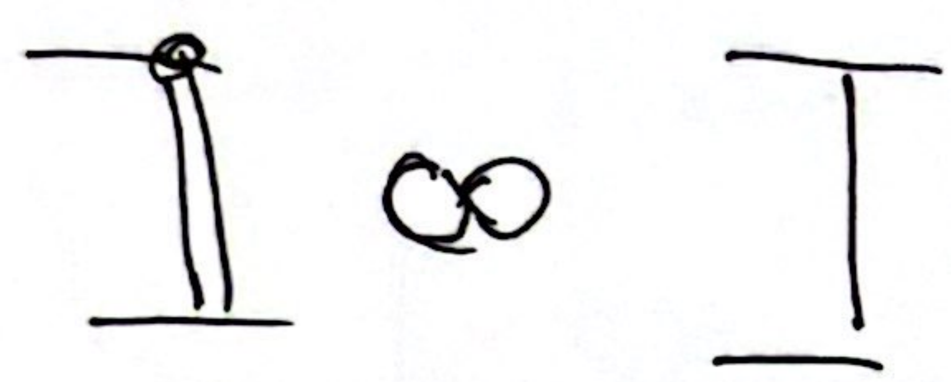
$4\times 3\times 2\times 1 = 24.$

$3\times 4\times 2\times 3 = 12\times 6$

②


$\cancel{4\times 3\times 4 \times 2\times 2}$

$\cancel{= 12\times 6}$

③


$3\times 3 = 9$

coordinate space

In S-matrix

trival part

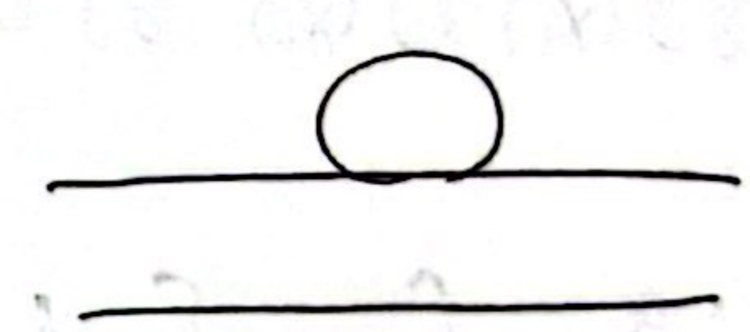
line. 

$\Delta = (x-y)$

Vertex $\times$ z $-ig.$

disconnected

symetry factor $S/4!$



non trivial

Connected diagram Generating functional.

connected

$Z[J] = e^{iW[J]}$ like free energy. X

$W[J] = -i\ln Z[J]$

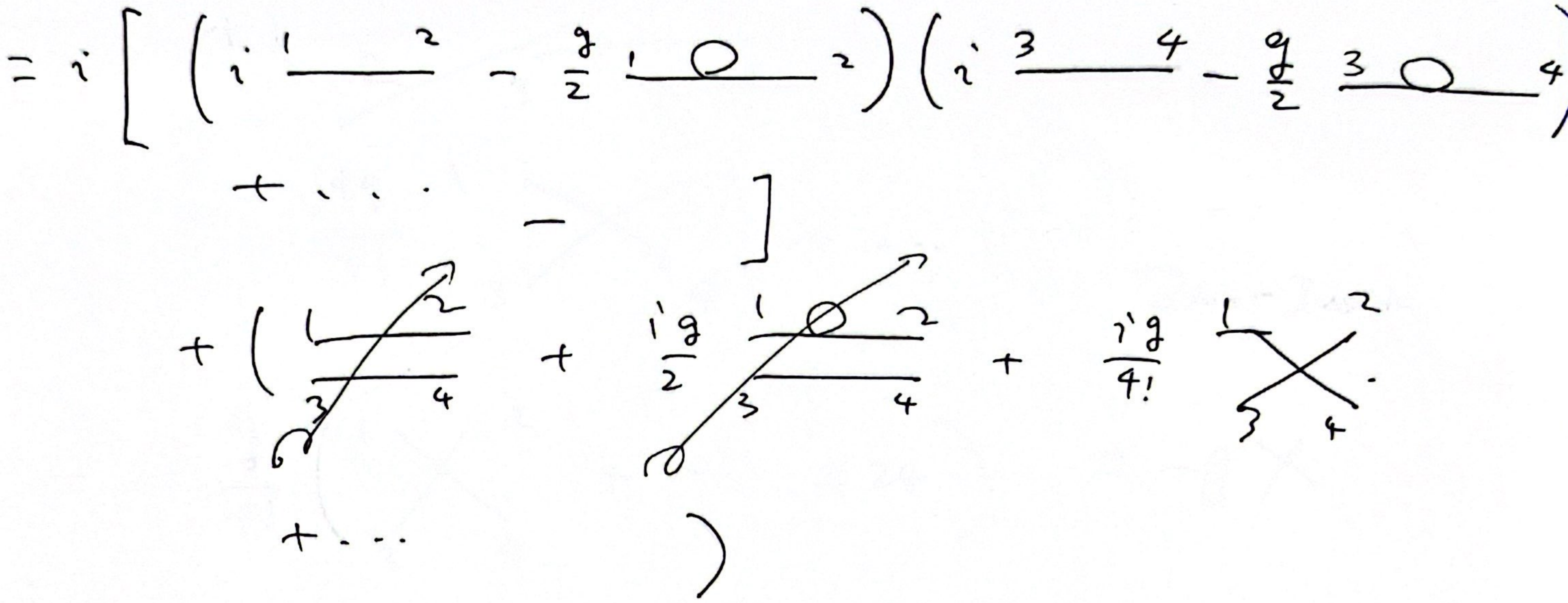
$$\frac{\delta^2 W}{\delta J(x_1)\delta J(x_2)} = \frac{i}{Z^2}\frac{\delta Z}{\delta J(x_1)}\frac{\delta Z}{\delta J(x_2)} - \frac{i}{Z}\frac{\delta^2 Z}{\delta J(x_1)\delta J(x_2)}$$

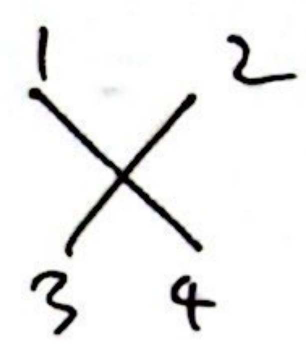
$$\bigcirc\Big|_{J=0} = -i\,\frac{\delta^2 Z}{\delta J(x_1)\,\delta J(x_2)}\Big|_{J=0} = i\,\tau(x_1, x_2)$$

$$\frac{\delta^4 W}{\delta J(x_1)\,\delta J(x_2)\,\delta J(x_3)\,\delta J(x_4)}\Big|_{J=0}$$

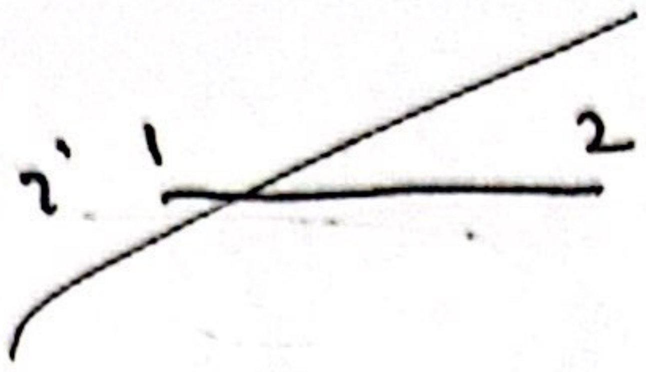
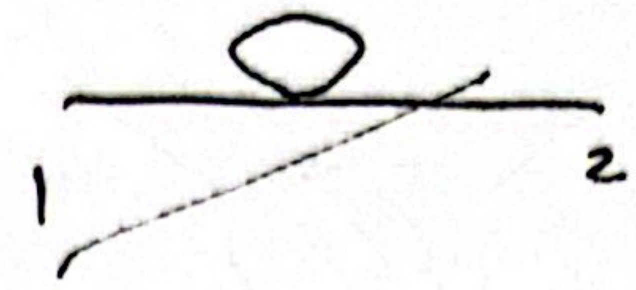
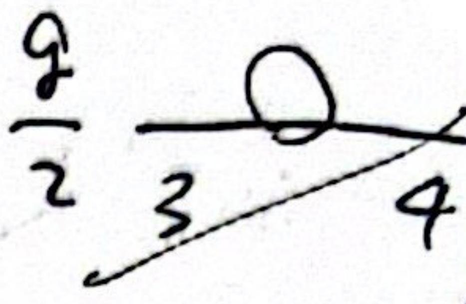
$$= i\big[\tau(x_1,x_2)\,\tau(x_3,x_4) + \tau(x_1,x_3)\,\tau(x_2,x_4)$$

$$+\,\tau(x_1,x_4)\,\tau(x_2,x_3) - \tau(x_1,x_2,x_3,x_4)\big]$$

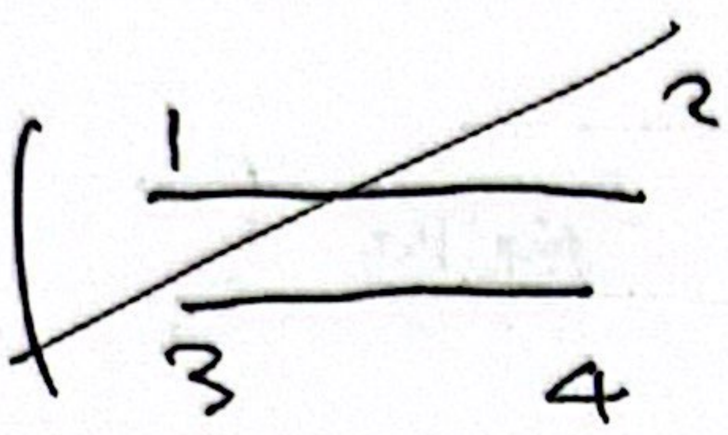


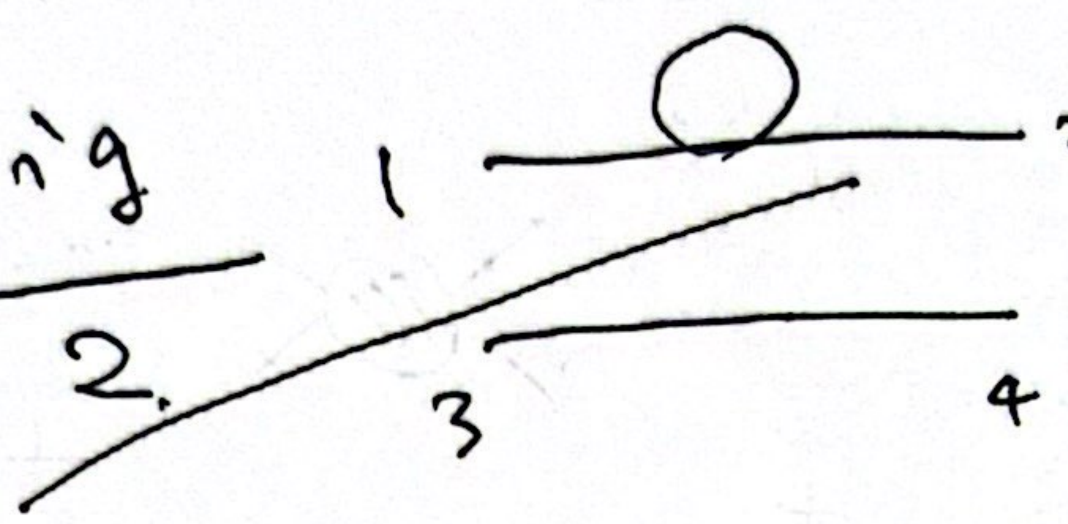
$= -\frac{g}{4!}\Big($  $+ \cdots \Big) = -g\,X$

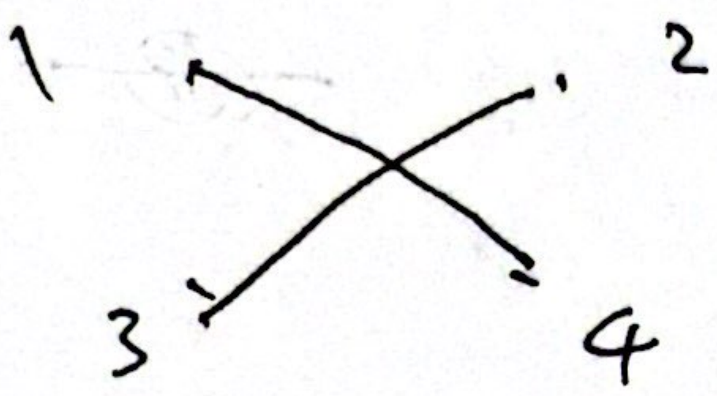
\# 24

$= i\Big[\Big(i$  $- \frac{g}{2}$  $\Big)\Big(i$  $- \frac{g}{2}$ 

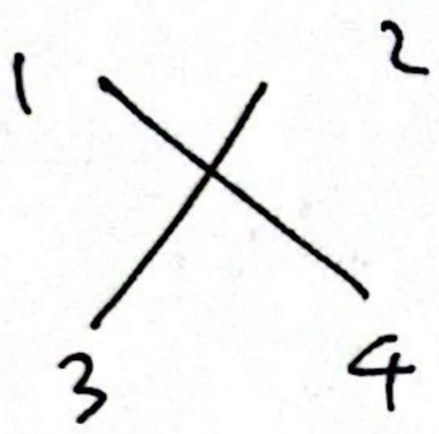
$+ \cdots + \cdots$

$+ \Big($  $+ \cdots \Big)$

$+ \Big(\frac{ig}{2}$  $+ \cdots \Big)$

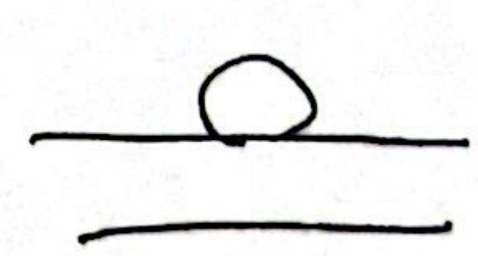
$+ \frac{ig}{4!}$  $+ \cdots$

Zinn-Justin

$= -\frac{g}{4!}\Big($  $+ \cdots \# 24 \Big) = -g\,X$

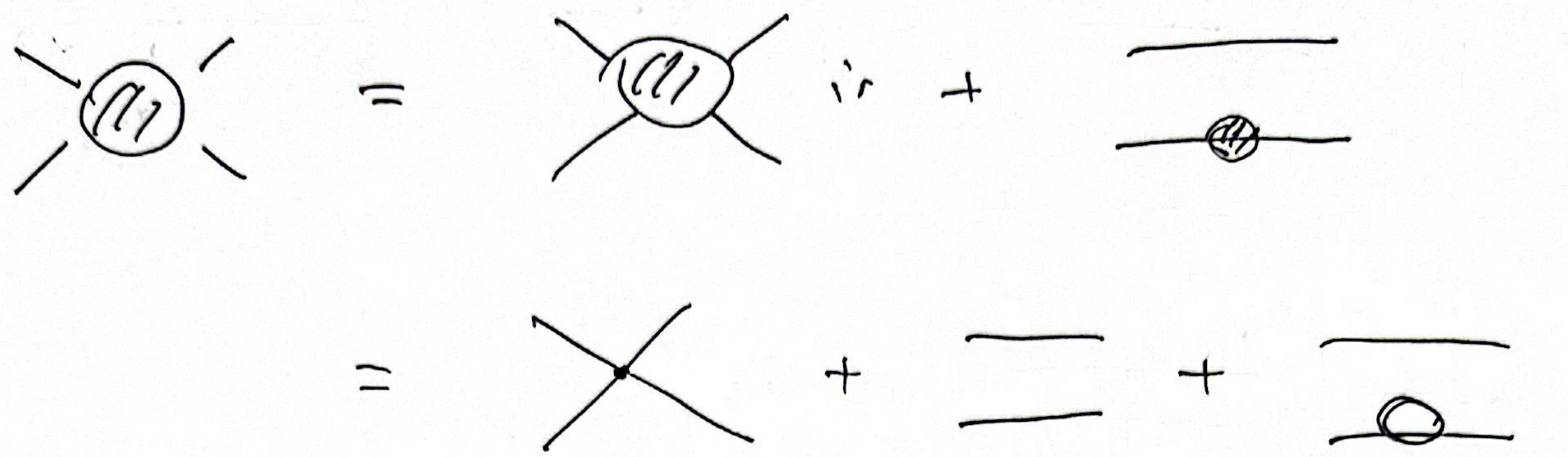
$$\tau(x_1, \cdots x_n) = \frac{1}{i^n} \frac{\delta^n Z}{\delta(x_1 \cdots x_n)}\Big|_{J=0}$$

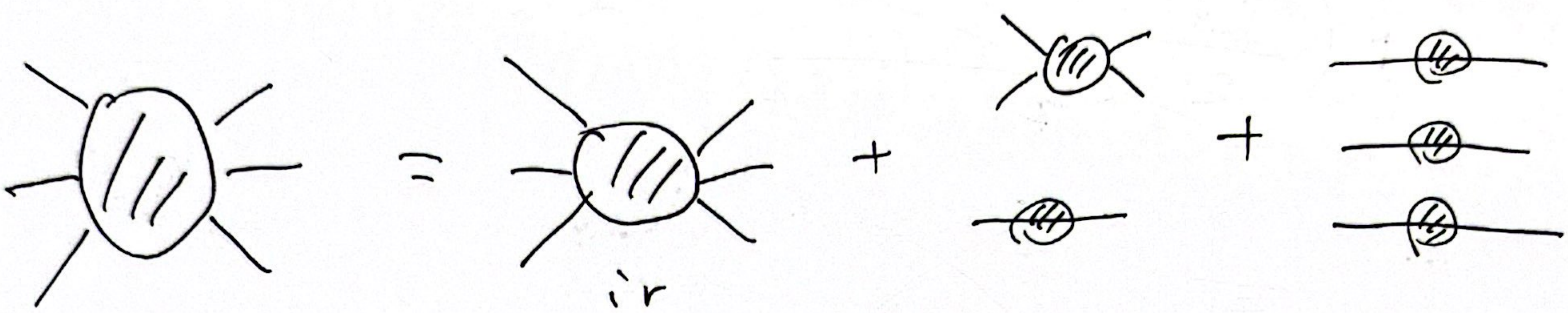
$$\phi(x_1, \cdots x_n) = \frac{1}{i^n} \frac{\delta^n W}{\delta(x_1, \cdots x_n)}\Big|_{J=0}$$ irreducible.

$\tau(x_1 \cdots x_4) = \underline{-ig\,X} - 3ig$  $- 3$ 

$$i\phi(x_1, \cdots x_4) = -ig\,X$$

$$\tau(x_1, \cdots x_4) = i\phi(x_1, \cdots x_4) - \sum_P \phi(x_{i_1}, x_{i_2})\,\phi(x_{i_3}, x_{i_4})$$


ir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