Free particle Green's functions.

$$Z_0[J] = N \exp\left[-\tfrac{i}{2}\int J(x)\,\Delta(x-y)J(y)\,dx\,dy\right]$$

$$= N\left\{1 - \tfrac{i}{2}\int J(x)\,\Delta_F(x-y)\,J(y)\,dx\,dy + \tfrac{1}{2!}\left(\tfrac{i}{2}\right)^2\left[\int J(x)\,\Delta(x-y)\,J(y)\,dx\,dy\right]^2 + \cdots\right\}$$

$$J(x) = \int J(p)\,e^{-ipx}\,d^4p$$

$$\Delta_F(x) = \frac{1}{(2\pi)^4}\int d^4k\,\frac{e^{-ikx}}{k^2-m^2+i\epsilon}$$

$$Z_0^{(1)}[J] = -\tfrac{i}{2}\int J(x)\,\Delta_F(x-y)\,J(y)\,dx\,dy$$

$$= -\frac{i}{2(2\pi)^4}\int \frac{J(p_1)\,e^{-ip_1x}\,e^{-ik(x-y)}\,e^{-ip_2y}\,J(p_2)}{k^2-m^2+i\epsilon}\,dp_1\,dp_2\,dx\,dy\,dk$$

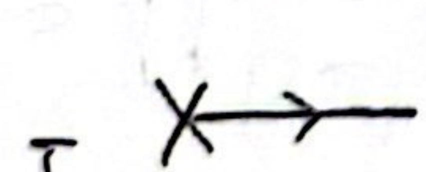
$$= -\frac{i}{2(2\pi)^4}\int \frac{J(p_1)\,e^{-i(p_1+k)x}\,e^{-i(p_2-k)y}\,J(p_2)}{k^2-m^2+i\epsilon}\,dp_1\,dp_2\,dk\,dx\,dy$$

$$= -\frac{i}{2}(2\pi)^4\int \frac{J(-k)\,J(k)}{k^2-m^2+i\epsilon}\,dk.$$

① vacuum - vacuum amplitude

$$\frac{1}{(2\pi)^4}\,\frac{i}{k^2-m^2+i\epsilon}$$

$\frac{1}{2}$



$i(2\pi)^4 J(p)$

$i(2\pi)^4 J(-p)$

$$Z_0[J] = 1 + \frac{1}{2}\, \times\!\!\longrightarrow\!\!\times + \frac{1}{2!}\left(\frac{1}{2}\right)^2 \begin{matrix}\times\!\!\longrightarrow\!\!\times \\ \times\!\!\longrightarrow\!\!\times\end{matrix} + \frac{1}{3!}\left(\frac{1}{2}\right)^3 \begin{matrix}\times\!\!\longrightarrow\!\!\times \\ \times\!\!\longrightarrow\!\!\times \\ \times\!\!\longrightarrow\!\!\times\end{matrix}$$

field theory $\longrightarrow$ many particle theory

Normalization.

$$Z_0[J=0] = 1$$

$$Z_0[J] = \langle 0,\infty | 0,-\infty \rangle^J$$

$$= \frac{\int D\phi \exp\{-i\int [\frac{1}{2}\phi(\Box + m^2 - i\epsilon)\phi - \phi J]\, dx}{\int D\phi \exp[-i\int \frac{1}{2}\phi(\Box + m^2 - i\epsilon)\phi\, dx}.$$

$$= \exp\left(-\frac{i}{2}\int J(x)\Delta(x-y)J(y)\, dx\, dy\right).$$

Generating functional

$$F(y_1, \cdots y_k) = \sum_{n=0}^{\infty}\sum_{i_1=0}^{k}\cdots\sum_{i_{n-1}=0}^{k}\sum_{i_n=0}^{k} \frac{1}{n!} T_n(i_1, \cdots i_n)\, y_{i_1}\cdots y_{i_n}$$

$$T_n(i_1, \cdots i_n) \cancel{\mathcal{T}_n} = \frac{\partial^n F(y_1, \cdots, y_k)}{\partial y_{i_1}\cdots \partial y_{i_{n-1}}\, \partial y_{i_n}}\Bigg|_{y=0}$$

$$F[y] = \sum_{n=0}^{\infty}\int dx_1 \cdots dx_n \frac{1}{n!} T_n(x_1, \cdots, x_n)\, y(x_1)\cdots y(x_n)$$

$$T_n(x_1, \cdots, x_n) = \frac{\delta}{\delta y(x_1)}\frac{\delta}{\delta y(x_2)}\cdots\frac{\delta}{\delta y(x_n)} F[y]\Bigg|_{y=0}$$

$$\tau(x_1, \cdots, x_n) = \frac{1}{i^n}\frac{\delta^n Z_0[J]}{\delta J(x_1)\cdots\delta J(x_n)}\Bigg|_{J=0}$$

$$= \langle 0 | T[\phi(x_1)\cdots\phi(x_n)] | 0 \rangle$$

$$Z_0[J] = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int dx_1 \cdots dx_n \, J(x_1) \cdots J(x_n) \, \tau(x_1, \cdots, x_n)$$

$$\tau(x,y) = - \frac{\delta^2 Z_0[J]}{\delta J(x) \, \delta J(y)} = i \, \Delta_F(x-y)$$

$$= \langle 0 | T[\phi(x)\phi(y)] | 0 \rangle$$

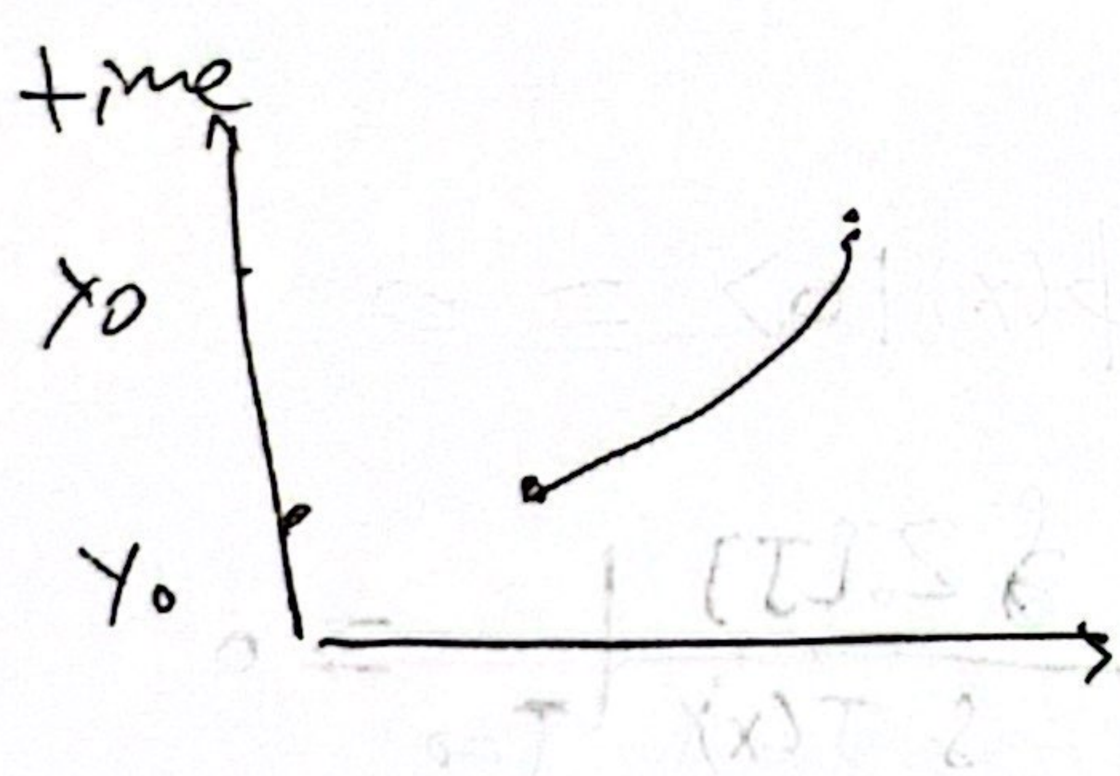
$$= \langle 0 | \, \theta(x_0 - y_0) \phi(x) \phi(y) + \phi(y_0 - x_0) \, \phi(y) \phi(x) | 0 \rangle$$

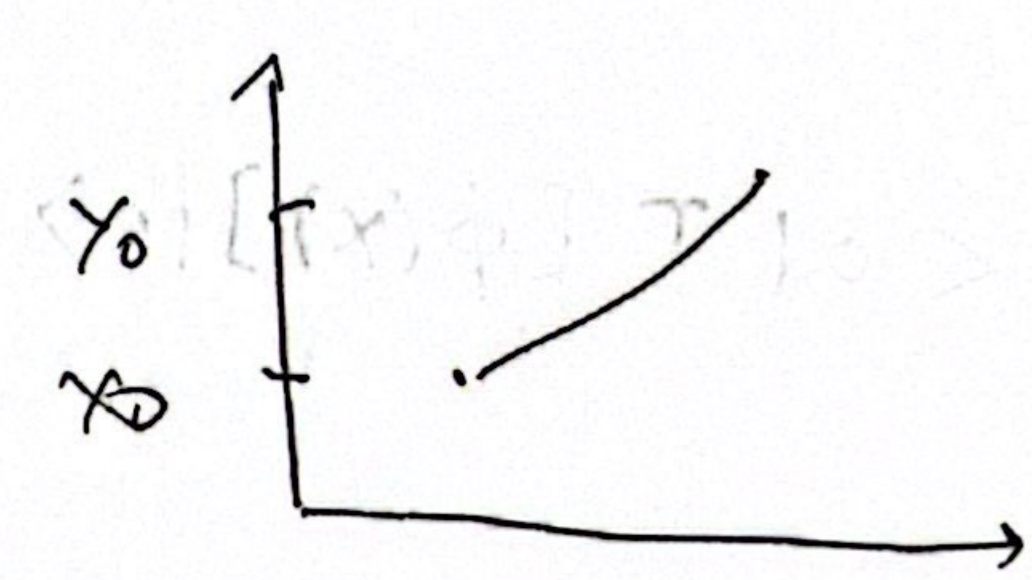
$$\phi(x) = \phi^+(x) + \phi^-(x)$$

$$= \int d^3\tilde{k} \, \underbrace{f_k(x)}_{e^{-ikx}} a(k) + \int d^3\tilde{k} \, f_k^*(x) \, a^\dagger(k)$$

$$\tau(x,y) = \theta(x_0 - y_0) \, \langle 0 | \, \phi^+(x) \phi^-(y) | 0 \rangle +$$

$$\theta(y_0 - x) \, \langle 0 | \phi^+(y) \, \phi^-(x) | 0 \rangle$$





$$= \int \frac{d^3k \, d^3k'}{(2\pi)^3 (2\omega_k 2\omega_{k'})^{1/2}} \Big[\theta(x_0 - y_0) \langle 0 | a(k) f_k(x) \, a^\dagger(k') f_{k'}^*(y) | 0 \rangle$$

$$+ \theta(y_0 - x_0) \, \langle 0 | a(k) f_k(y) \, a^\dagger(k') \, f_{k'}^*(x) | 0 \rangle \Big]$$

$$= \int \frac{d^3k \, d^3k'}{(2\pi)^6 \; 2\omega_k 2\omega_{k'}} \Big[\theta(x_0 - y_0) \, e^{-i(kx - k'y)} \langle 0 | a(k) a^\dagger(k') | 0 \rangle +$$

$$\theta(y_0 - x_0) \, e^{-i(ky - k'x)} \langle 0 | a(k) \, a^\dagger(k') | 0 \rangle \Big]$$

$$[a(k), a^{+}(k')] = (2\pi)^3\, 2w_k\, \delta^3(k-k')$$

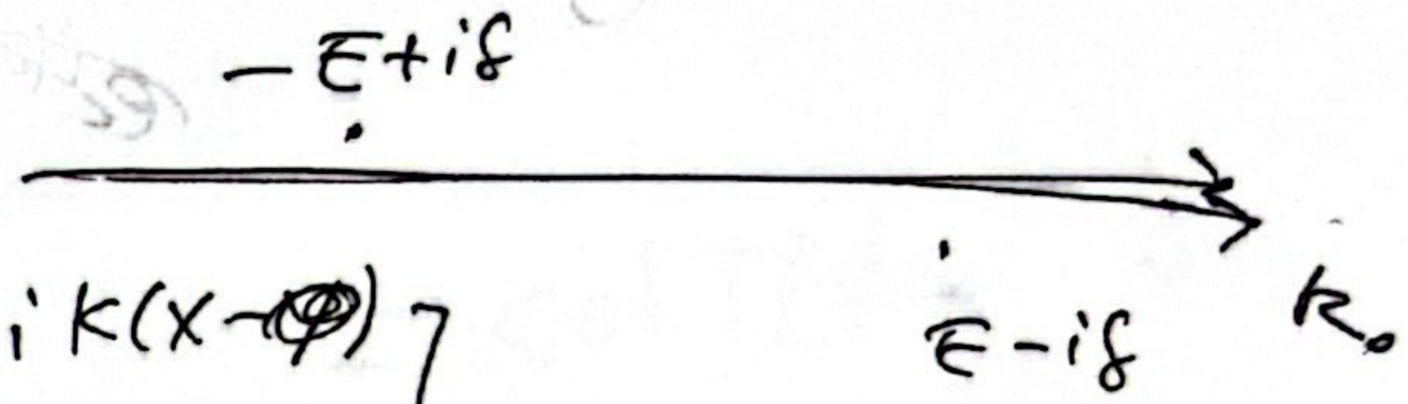
$$w_k = \sqrt{k^2+m^2}$$

$$\tau(x,y) = \int \frac{d^3k}{(2\pi)^3\, 2w_k} \Big[\theta(x_0-y_0)\, e^{-ik(x-y)} + \theta(y_0-x_0)\, e^{ik(x-y)} \Big]$$

$$\Delta_F(x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{k^2-m^2+i\epsilon}$$

$$= \int \frac{d^3k\, dk_0}{(2\pi)^4} \frac{e^{-ikx}}{2w_k} \left(\frac{1}{k_0 - w_k + i\delta} - \frac{1}{k_0 + w_k - i\delta} \right)$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{x}}}{2w_k} \Big[\theta(x_0)(-i)\, e^{-iw_k x_0} - \theta(-x_0)\, i\, e^{iw_k x_0} \Big]$$



$$= -i \int \frac{d^3k}{(2\pi)^3} \Big[\theta(x_0)\, e^{-ikx} + \theta(-x_0)\, e^{ik(x)} \Big]$$

$$\tau(x,y) = +i\, \Delta_F(x-y)$$

1 - point

$$\tau(x) = \langle 0 | T[\phi(x)] | 0 \rangle = \langle 0 | \phi(x) | 0 \rangle = 0$$

$$= \frac{1}{i} \frac{\delta Z_0[J]}{\delta J(x)} \Big|_{J=0} = 0$$

3 - point

$$Z_0[J] = \exp\left[-\frac{i}{2} \int J(x)\, \Delta_F(x-y)\, J(y)\, dx\, dy \right]$$

$$= \sum_n \frac{1}{n!} \left(-\frac{i}{2}\right)^n \left(\int J(x)\, \Delta_F(x-y)\, J(y)\, dx\, dy \right)^n$$

$$\tau(x_1,x_2,x_3) = \frac{1}{i} \frac{\delta}{\delta J(x_1)} \frac{1}{i} \frac{\delta}{\delta J(x_2)} \frac{1}{i} \frac{\delta}{\delta J(x_3)} Z_0[J] \Big|_{J=0} = 0$$

$$-\frac{1}{2}\frac{1}{4}\int J(x)\,\Delta_F(x-y)\,J(y)\,J(x')\,\Delta_F(x'-y')\,J(y')\,dx\,dy\,dx'\,dy'$$

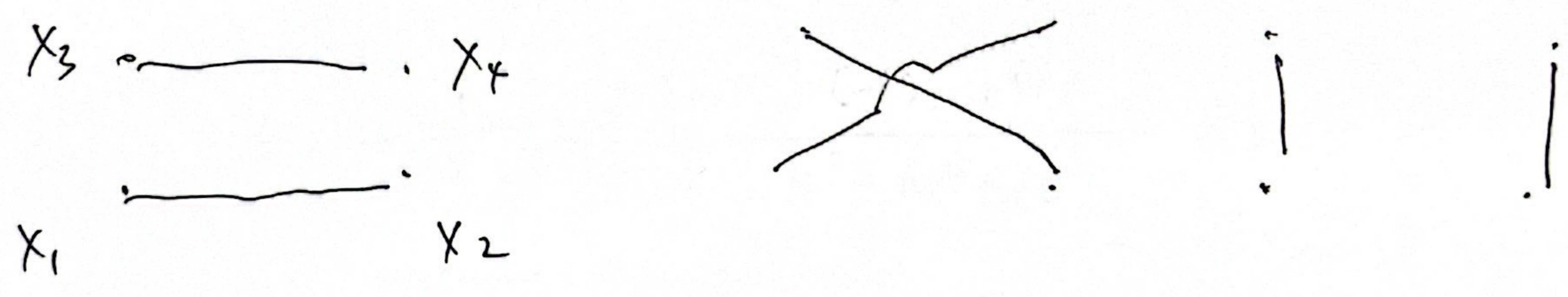
$\tau(x_1, x_2, x_3, x_4)$

$$\begin{array}{cccc} x & y & x' & y' \\ \vdots & \vdots & \vdots & \vdots \\ x_1 & x_2 & x_3 & x_4 \end{array}$$

$$= -\frac{1}{2}\frac{1}{4}\cdot\frac{1}{i^4}\cdot 8$$

$$\Big(\Delta_F(x_1-x_2)\,\Delta_F(x_3-x_4) + \Delta_F(x_2-x_3)\,\Delta_F(x_1-x_4) + \Delta_F(x_1-x_3)\,\Delta_F(x_2-x_4) \Big)$$

$$= -\Big(\Delta_F(x_1-x_2)\,\Delta_F(x_3-x_4) + \Delta_F(x_2-x_3)\,\Delta_F(x_1-x_4) + \Delta_F(x_1-x_3)\,\Delta_F(x_2-x_4) \Big) = \langle 0|T[\phi(x_1)\cdots\phi(x_4)]|0\rangle$$



$$\tau(x_1, \cdots x_{2n+1}) = 0$$

$$\tau(x_1, \cdots, x_{2n}) = \sum_{\text{perms}} \tau(x_{p_1}, x_{p_2}) \cdots \tau(x_{p_{2n-1}}, x_{p_{2n}})$$

Wick's theorem

$$\tau(x, y) = i\,\Delta_F(x-y)$$