Color decomposition

2024年5月19日 19:55

Amplitude in gauge theory $\to \sum$ partial Trace $\times$ partial amplitude

$Tr(T^aT^bT^c)$ $A(1.2.3.4)$. $\to$ kinematics. color ✗

gauge invariant   gauge invariant ✓

Fierz Identity. $SU(N_c)$

$$\sum_{a=1}^{N_c^2-1}(t^a)_{i_1j_1}(t^a)_{i_2j_2} = \frac{1}{2}\left(\delta_{i_1j_2}\delta_{i_2j_1} - \frac{1}{N_c}\delta_{i_1j_1}\delta_{i_2j_2}\right)$$

$$\tilde{T}^a = \sqrt{2}t^a,\ a=1\ldots N_c^2-1 \Rightarrow \sum_a(\tilde{T}^a)_{i_1j_1}(\tilde{T}^a)_{i_2j_2} = \delta_{i_1j_2}\delta_{i_2j_1} - \underline{\frac{1}{N_c}\delta_{i_1j_1}\delta_{i_2j_2}}.$$

N_c^2-1 : from $SU(N_c)$. $U(N_c)$. : special -1 DoF.

$\Rightarrow$ fix $SU(N_c) \to U(N_c)$. need 1 more generator $\tilde{T}^{N_c^2}$

define : $(\tilde{T}^{N_c^2})_{ij} = \frac{1}{\sqrt{N_c}}\delta_{ij}$ $\Rightarrow$ $\begin{cases}[\tilde{T}^{N_c^2},\tilde{T}^a] = 0. \quad a=1,\ldots N_c^2-1. \\ \sum_{a=1}^{N_c^2}(\tilde{T}^a)_{i_1j_1}(\tilde{T}^a)_{i_2j_2} = \delta_{i_1j_2}\delta_{i_2j_1} \quad \text{(Fierz. id.)}.\end{cases}$

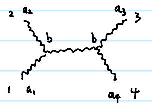
$$(\tilde{T}^{N_c^2})_{i_1j_1}(\tilde{T}^{N_c^2})_{i_2j_2} = \underline{\frac{1}{N_c}\delta_{i_1j_1}\delta_{i_2j_2}}.$$

cancel.

structure constant.

$$f^{abc} = \frac{-i}{\sqrt{2}}\left\{tr(\tilde{T}^a\tilde{T}^b\tilde{T}^c) - tr(\tilde{T}^a\tilde{T}^c\tilde{T}^b)\right\}$$

example :



$\sim \boxed{\sum_{b=1}^{N_c^2-1} f^{a_1a_2b}f^{ba_3a_4}}$ (by def.)

$= tr(\ )tr(\ ) + \cdots + \cdots$

one term is : $-\frac{1}{2}tr(\tilde{T}^{a_1}\tilde{T}^{a_2}\tilde{T}^b)tr(\tilde{T}^b\tilde{T}^{a_3}\tilde{T}^{a_4})\ldots$

$$\sum_b^{N_c^2-1} tr(\tilde{T}^{a_1}\tilde{T}^{a_2}\tilde{T}^b)tr(\tilde{T}^b\tilde{T}^{a_3}\tilde{T}^{a_4})$$

$$= \sum_b^{N_c^2-1}\tilde{T}^{a_1}_{ij}\tilde{T}^{a_2}_{jk}\underline{\tilde{T}^b_{ki}\tilde{T}^b_{lm}}\tilde{T}^{a_3}_{mn}\tilde{T}^{a_4}_{nl}$$

$$= \tilde{T}^{a_1}_{ij}\tilde{T}^{a_2}_{jk}\tilde{T}^{a_3}_{mn}\tilde{T}^{a_4}_{nl}\left[\delta_{km}\delta_{il} - \frac{1}{N_c}\delta_{ki}\delta_{ml}\right]$$

$$= tr(\tilde{T}^{a_1}\tilde{T}^{a_2}\tilde{T}^{a_3}\tilde{T}^{a_4}) - \frac{1}{N_c}tr(\tilde{T}^{a_1}\tilde{T}^{a_2})tr(\tilde{T}^{a_3}\tilde{T}^{a_4}).$$

$$\Rightarrow A^{tree} = \sum_{\sigma\in S_n/Z_n} tr(\tilde{T}^{a_{\sigma(1)}}\tilde{T}^{a_{\sigma(2)}}\ldots\tilde{T}^{a_{\sigma(n)}})\,A^{partial}(\sigma(1).\sigma(2)\ldots\sigma(n)).$$

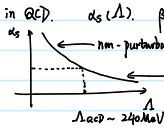
color decomposition.

- Where is the $\frac{1}{N_c}tr^2$ term ?
- cancel in tree level. double trace / triple trace …
  $\Rightarrow$ loop diagram

Renormalization :

RGE : running coupling : in QED. $\alpha_e(\Lambda)$ is not a constant β-function >0 $\Rightarrow \Lambda\nearrow$ $\alpha_e\nearrow$. Landau pole $\Lambda \sim 10^{200}eV$.

in QCD. $\alpha_s(\Lambda)$. β-function <0. $\Lambda\nearrow$ $\alpha_s\searrow$. $\Rightarrow \Lambda_{QCD}$.



non-perturbative. ? lattice QCD.

perturbative . Fienman diagram.

hadronization : 强子化 . no free quark.

no free gluon.

color confinement (色禁闭)

proton mass problem.

partial amplitude : 4-gluons : how many different partial amplitude ?

$A(1234)$ $A(1342)$ $A(1423)$ $A(1243)$ $A(1324)$ $A(1432)$.

fix 1 at $\sigma(1)$ $\Rightarrow S_n/Z_n$ these 6 terms are not independent.

let $\tilde{T}^1 = \frac{1}{\sqrt{N}}I$. 1: "photon"

$\Rightarrow$ $tr(\tilde{T}^2\tilde{T}^3\tilde{T}^4)[A(1234)+A(1423)+A(1342)]$

$+tr(\tilde{T}^2\tilde{T}^4\tilde{T}^3)[A(1243)+A(1324)+A(1432)] = 0.$

(1) $A(1234)+A(1423)+A(1342) = 0$

(2) $A(1243)+A(1324)+A(1432) = 0$

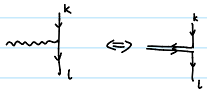
} from Kleiss-Kuijf relations.

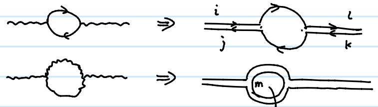
- Lance Dixon / Yu-ting Huang $\to$ spinor helicity notation.
- 't Hooft Large N_c Limit.

g: coupling constant   introduce $\lambda \equiv g^2N_c$. fix λ : $N_c\to\infty$

$$\langle A_{\mu.ij}(x)A_{\nu.kl}(y)\rangle = D_{\mu\nu}(x-y)\cdot\frac{1}{2}\left[\delta_{il}\delta_{jk} - \underline{\frac{1}{N_c}\delta_{ij}\delta_{kl}}\right].$$

vanish

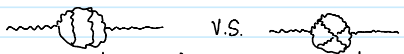




$\sim g^2\delta_{il}\delta_{kj}$. $\to$ suppressed

$\sim g^2N_c\delta_{il}\delta_{kl}$. $\to$ major

$\Sigma m = N_c$ (quark only has N_c different color indices)



planar diagram (N_c^2) $\gg$ non-planar diagram (1)

V.S.

QCD : $N_c = 3$ (is enough).

SYM ($N=4$) consider 't Hooft limit.