BRST and Slavnov-Taylor identity

$$Z = N \int DA_\mu D\bar{\eta} D\eta \ \exp\left(i \int \mathcal{L}_{eff} d^4x\right)$$

$$\mathcal{L}_{eff} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} - \frac{1}{2\xi}(\partial \cdot A)^2 - \underline{\eta^a (\partial^\mu D^{ab}_\mu) \eta^b}.$$

$$-\bar{\eta}^a (\delta^{ab} \Box - g f^{abc} \partial^\mu A^c_\mu - g f^{abc} A^c_\mu \partial^\mu) \eta^b$$

$$= \partial^\mu \bar{\eta}^a (\partial_\mu \eta^a + g f^{abc} A^b_\mu \eta^c) + \cdots$$

$$= -\bar{\eta}^a \partial^\mu D^{ab}_\mu \eta^b + \cdots$$

$$\delta A^a_\mu = \frac{1}{g} \partial_\mu \Lambda^a + \underline{f^{abc} A^b_\mu \Lambda^c} = \frac{1}{g}(D^{ab}_\mu \Lambda^b)$$

choose $\Lambda^a = -\eta^a \underset{\uparrow}{\lambda}$

Grassmann number

$$\delta A^a_\mu = -\frac{1}{g}(D_\mu \eta^a)\lambda$$

$$\delta \eta^a = -\frac{1}{2} f^{abc} \eta^b \eta^c \lambda \qquad \Rightarrow \mathcal{L}_{eff} \text{ invariant}$$

$$\delta \bar{\eta}^a = -\frac{1}{\xi g}(\partial \cdot A^a)\lambda$$

$$\delta \mathcal{L}_{GF} = \cancel{\frac{1}{\xi}(\partial \cdot A^a) \frac{1}{g}(\partial^\nu D_\nu \eta^a)\lambda}$$

$$\delta \mathcal{L}_{FPG} = -(\delta \bar{\eta}^a)\partial^\mu D_\mu \eta^a - \bar{\eta}^a \partial^\mu (\delta D_\mu \eta^a) = 0$$

$$= \cancel{-\frac{1}{\xi g}(\partial^\mu A^a_\mu)(\partial^\nu D_\nu \eta^a)\lambda} - \cdots$$

$$\delta(D_\mu \eta^a) = \delta(\partial_\mu \eta^a + g f^{abc} A^b_\mu \eta^c)$$

$$= \partial_\mu(\delta \eta^a) + g f^{abc}(\delta A^b_\mu)\eta^c + g f^{abc} A^b_\mu \delta \eta^c.$$

$$= -\frac{1}{2} f^{abc}\,\partial_\mu(\eta^b\eta^c)\lambda - f^{abc}\left[\partial_\mu\eta^b + g f^{bmn} A_\mu^m \eta^n\right]\lambda\eta^c$$

$$+ g f^{abc} f^{cmn} A_\mu^b \left(-\frac{1}{2}\right)\eta^m\eta^n\lambda$$

$$= \cancel{-f^{abc}(\partial_\mu\eta^b)(\eta^c\lambda + \lambda\eta^c)} + g f^{abc} f^{bmn} A_\mu^m \eta^n\eta^c\lambda$$

$$- \frac{1}{2} g f^{abc} f^{cmn} A_\mu^b \eta^m\eta^n\lambda$$

$$= 0$$

$$\delta^2(A_\mu^a) = 0 \qquad \delta^2(\eta^a) = 0$$

Slavnov – Taylor identity.

$$Z[s,x,y;u,v] = \int D\bar\eta\, D\eta\, DA_\mu\, e^{i\int \mathcal{L}_{tot}\, d^4x}$$

$$\mathcal{L}_{tot} = \mathcal{L}_{eff} + s_\mu^a A^{a\mu} + x^a\eta^a + y^a\bar\eta^a + u_\mu^a\overbrace{\left(\frac{1}{g}D^\mu\eta\right)^a}^{\delta A_\mu} + v^a\overbrace{\left(-\frac{1}{2}f^{abc}\eta^b\eta^c\right)}^{\delta\eta}$$

$$J = J\left(\frac{A_\mu^a(x) + \delta A_\mu^a,\ \eta^a + \delta\eta^a(x),\ \bar\eta^a(x) + \delta\bar\eta^a(x)}{A_\mu^b(y),\ \eta^b(y),\ \bar\eta^b(y)}\right)$$

$$= \delta_{ab}\,\delta(x-y)\ ;$$

$$Z = \int DA_\mu\, D\bar\eta\, D\eta\, \exp\left\{i\left[S + \int dx\,(s_\mu^a\delta A^{\mu a} + x^a\delta\eta^a + y^a\delta\bar\eta^a)\right]\right\}$$

$$\int DA_\mu\, D\bar\eta\, D\eta\, e^{iS}\int dx\,(s_\mu^a\delta A^{\mu a} + x^a\delta\eta^a + y^a\delta\bar\eta^a) = 0$$

$$\lambda\int dx\left\{s_\mu^a\frac{\delta Z}{\delta u_\mu^a(x)} + x^a\frac{\delta Z}{\delta v^a(x)} - \frac{1}{\xi g}y^a\left[\partial_\mu\frac{\delta Z}{\delta s_\mu^a(x)}\right]\right\} = 0$$

$$\int dx \left[\zeta^a_\mu \frac{\delta W}{\delta u^a_\mu} + \chi^a \frac{\delta W}{\delta v^a} - \frac{1}{\xi g} y^a \left(\partial_\mu \frac{\partial W}{\delta \zeta^a_\mu} \right) \right] = 0$$

$$W[\zeta, \chi, y ; u, v] = \Gamma[A, \eta, \bar{\eta} ; u, v] + \int dx \left(\zeta^a_\mu A^{\mu a} + \chi^a \eta^a + y^a \bar{\eta}^a \right)$$

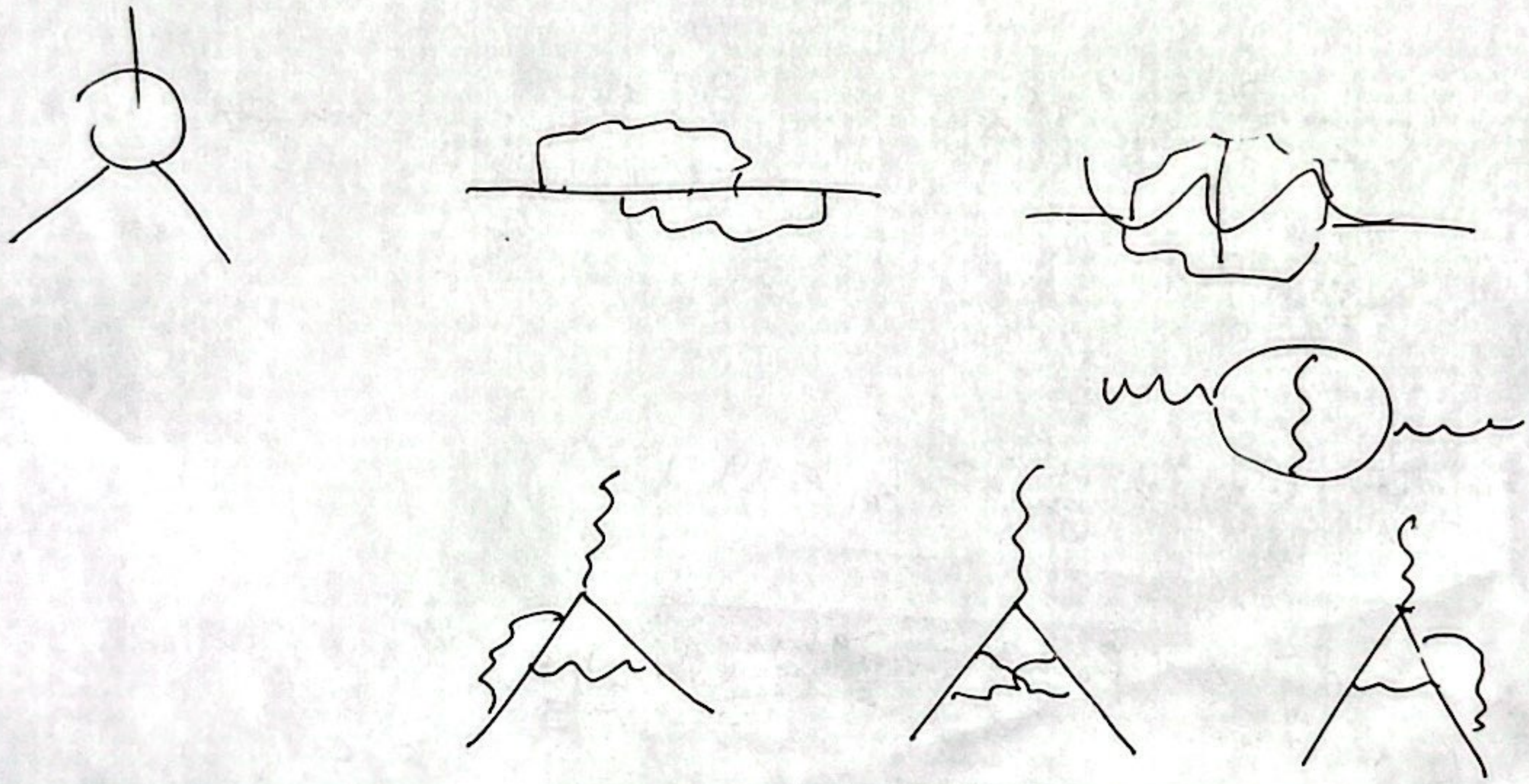
$$\zeta^a_\mu = - \frac{\delta \Gamma}{\delta A^{\mu a}}, \quad \chi^a = - \frac{\delta \Gamma}{\delta \eta^a}, \quad y^a = \frac{\delta \Gamma}{\delta \bar{\eta}^a}$$

$$\frac{\delta W}{\delta \zeta^a_\mu} = A^{\mu a}, \quad \frac{\delta W}{\delta u} = \frac{\delta \Gamma}{\delta u}, \quad \frac{\delta W}{\delta v} = \frac{\delta \Gamma}{\delta v}$$

$$\Rightarrow \int dx \, \frac{\delta \Gamma}{\delta A^a_\mu} \frac{\delta \Gamma}{\delta u^{\mu a}} + \frac{\delta \Gamma}{\delta \eta^a} \frac{\delta \Gamma}{\delta v^a} - \frac{1}{\xi g} (\partial \cdot A)^a \frac{\delta \Gamma}{\delta \bar{\eta}^a} \Big] = 0$$

$$\Gamma' = \Gamma + \frac{1}{2\xi} \int dx \, (\partial \cdot A)^{a\,2}$$

$$\int dx \left[\frac{\delta \Gamma'}{\delta u^{a\mu}} \frac{\delta \Gamma'}{\delta A^a_\mu} + \frac{\delta \Gamma'}{\delta v^a} \frac{\delta \Gamma}{\delta \eta^a} \right] = 0$$



Ghost & unitarity

$\langle m | S | n \rangle \qquad \langle m | n \rangle = \delta_{mn}, \quad \sum_n |m\rangle\langle m| = 1$

$$\sum_m |\langle m | S | n \rangle|^2 = 1 \Rightarrow S^\dagger S = 1$$

$S = 1 + iR \qquad S^\dagger S = 1$

$R - R^\dagger = i R^\dagger R \quad , \quad 2\,\mathrm{Im}\, R = R^\dagger R = R R^\dagger$

$\langle p_3 p_4 | \cdots | p_1 p_2 \rangle$

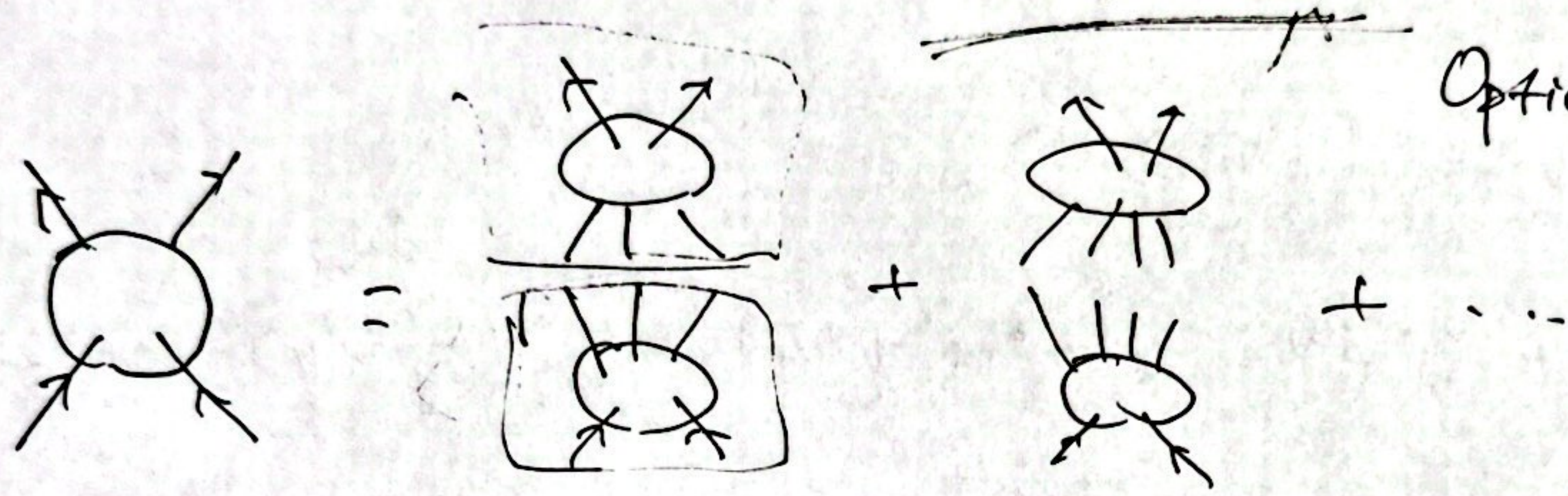
$$2\,\mathrm{Im} \langle p_3 p_4 | R | p_1 p_2 \rangle = \sum_n \langle p_3 p_4 | R | n \rangle \langle n | R | p_1 p_2 \rangle$$

$$R = \cancel{D}\ (2\pi)^4 \delta(P_f - P_i)\, T$$

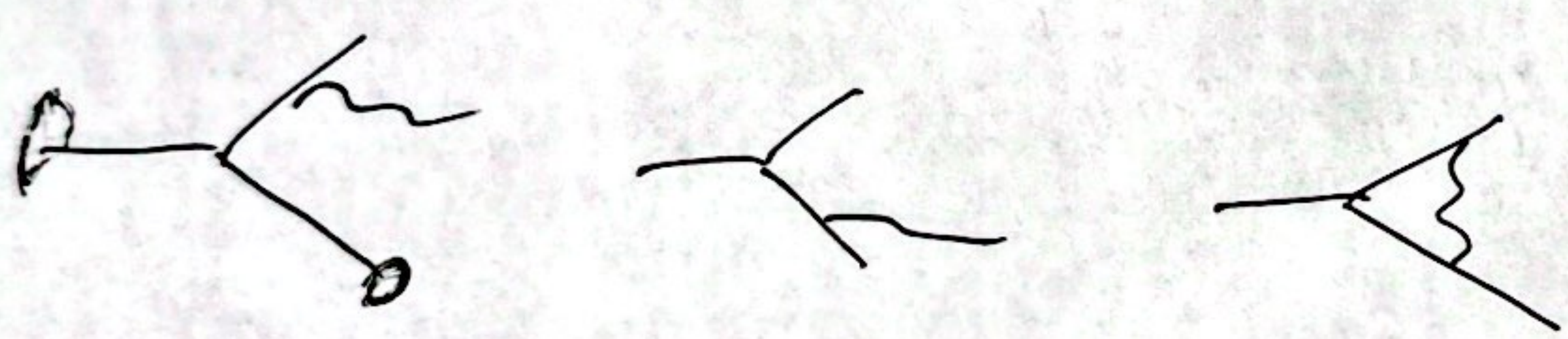
$$2\,\mathrm{Im} \langle p_3 p_4 | T | p_1 p_2 \rangle = \frac{1}{(2\pi)^2} \sum_n \frac{d^3k_1}{w_1} \frac{d^2k_2}{w_2} \cdots \delta^4(p_1 + p_2 - k_1 - \cdots - k_n)$$

$$\langle p_3 p_4 | T | k_1 \cdots k_n \rangle \langle p_1 p_2 | T^\dagger | k_1 \cdots k_n \rangle^*$$

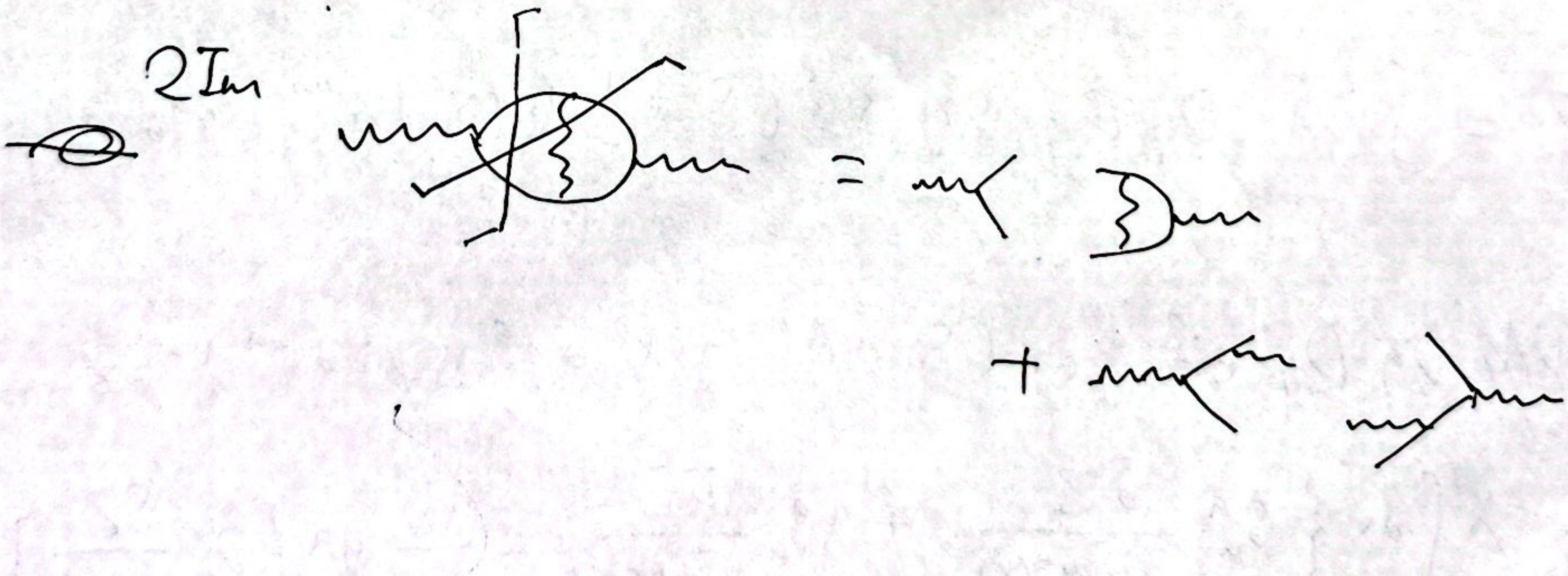
Optical theorem

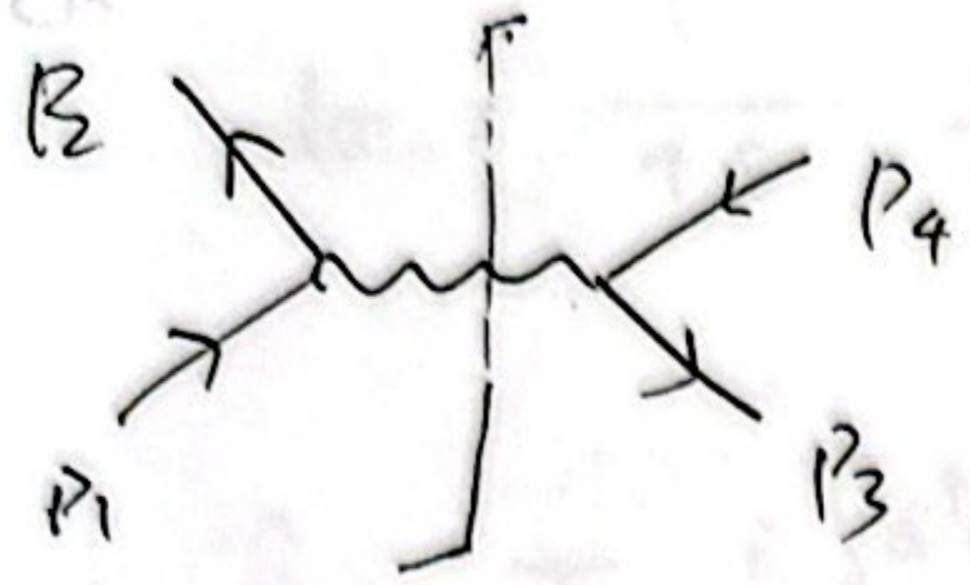
$2\,\mathrm{Im}$ 

$2\,\mathrm{Im}\, R_{ii} = \sum_a |R_{ai}|^2$



$2\,\mathrm{Im}$





$$A = \cancel{f(ie)^2}\,(-ie)^2\, \bar{v}(p_2)\gamma_\mu u(p_1) \frac{-ig^{\mu\nu}}{q^2+i\epsilon}\bar{u}(p_3)\gamma_\nu v(p_4)$$

$$\mathrm{Im}\,\frac{g^{\mu\nu}}{q^2+i\epsilon} = \left(\frac{g^{\mu\nu}}{q^2+i\epsilon} - \frac{g^{\mu\nu}}{q^2-i\epsilon}\right)\frac{1}{2i}$$

$$\sum_{\lambda=0}^{3} \varepsilon_\mu^{(\lambda)} \varepsilon_\nu^{(\lambda)} = -g_{\mu\nu} \qquad = -\frac{\epsilon}{q^4+\epsilon^2}\, g^{\mu\nu}$$

$$= -\pi g^{\mu\nu}\,\delta(q^2)\,\theta(q_0)$$

$$\delta(x) = \lim_{\epsilon\to 0} \frac{\epsilon}{x^2+\epsilon^2}\frac{1}{\pi}$$

$$\mathrm{Im}\,A = M^\mu \left[\sum_{\lambda=0}^{3} \varepsilon_\mu^{(\lambda)} \varepsilon_\nu^{(\lambda)}\right] M^{\nu\dagger}$$

$$M = \epsilon^\mu M_\mu$$

$$p^\mu M_\mu = 0$$

$$B = M^\mu \epsilon_\mu(q,\lambda)$$

$$BB^\dagger = M^\mu \sum_{\lambda=1}^{2} \varepsilon_\mu(q,\lambda)\,\varepsilon(q,\lambda)\, M^{\nu\dagger}$$

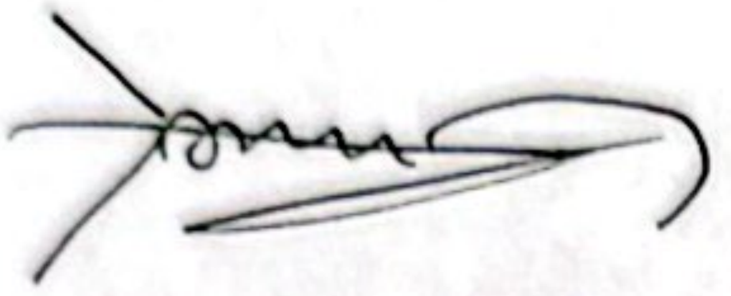
$$M^\mu \sum_{\lambda=0,3} \varepsilon_\mu^{(\lambda)}(q)\, \varepsilon_\nu^{(\lambda)}(q)\, M^{\nu\dagger} = 0$$

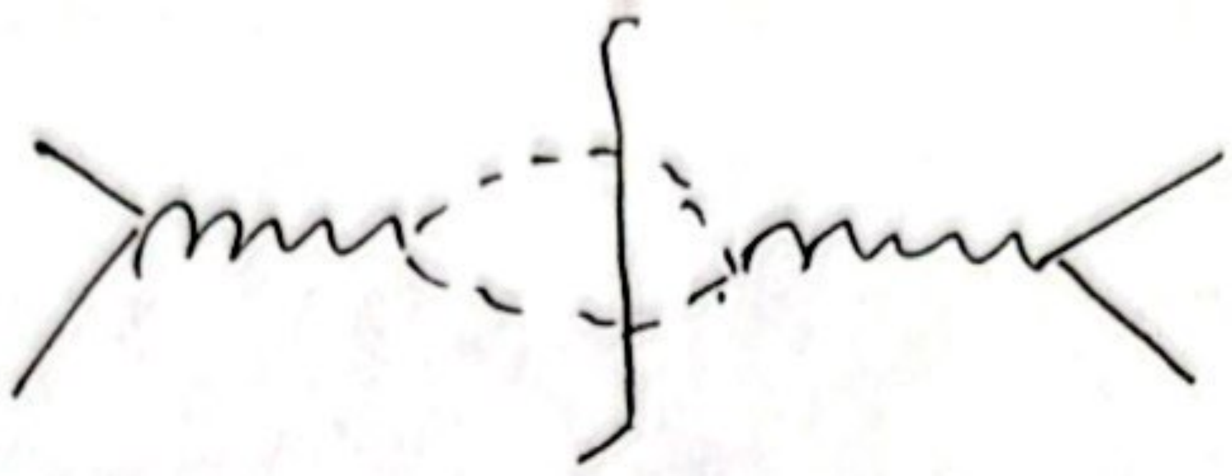
$$\sum_{\lambda=1}^{2} \varepsilon_\mu^{(\lambda)} \varepsilon_\nu^{(\lambda)} = -g^{\mu\nu} - \frac{t^2}{(q\cdot t)^2} q_\mu q_\nu + \frac{q_\mu t_\nu + t_\mu q_\nu}{(q\cdot t)}$$

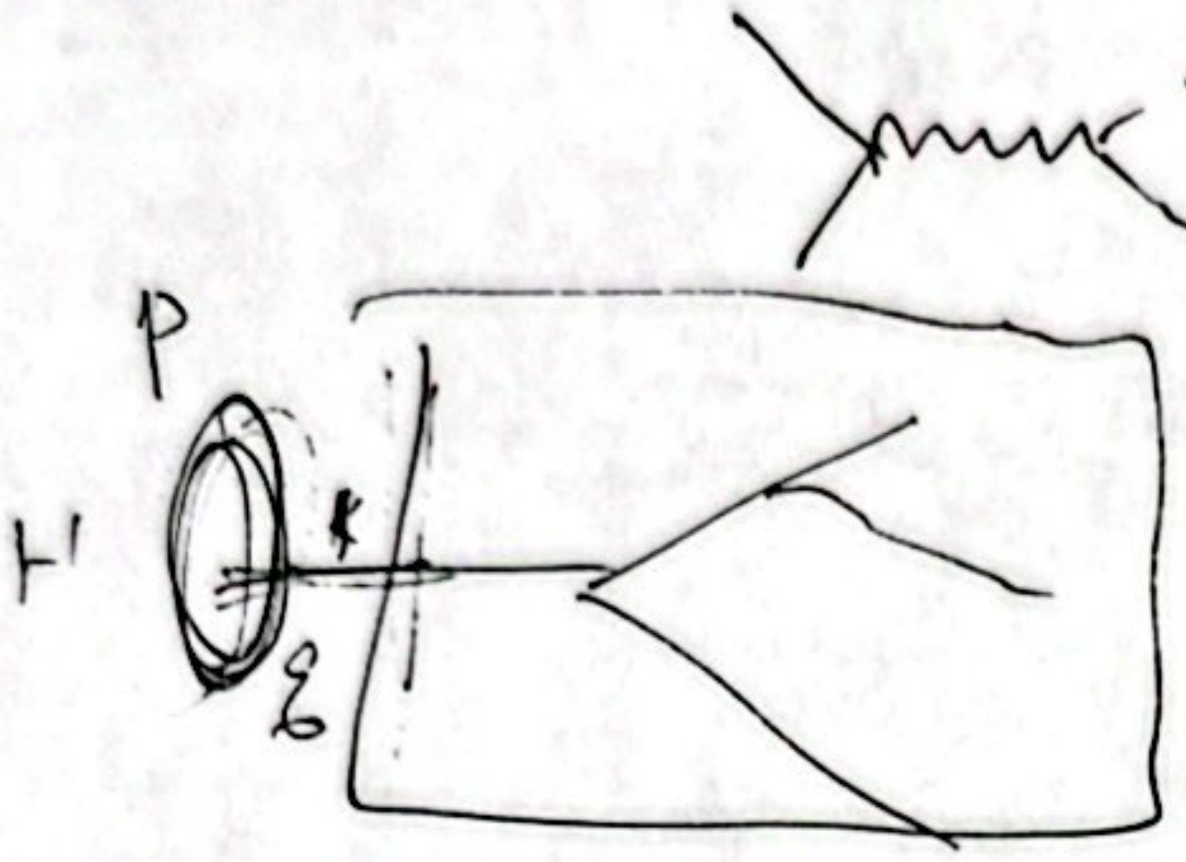
$$q^\mu M_\mu = 0$$

$$\epsilon^\mu M_\mu . \qquad \epsilon_\mu \to \epsilon_\mu + \alpha q_\mu$$

$$\Rightarrow \quad q^\mu M_\mu = 0$$







$$H(p) + A \to X$$

$$\sigma(H(p)+A\to X) = \sum_q \int_0^1 dx\, f_{q/H}(x)\, \hat{\sigma}(q+A\to X) + O\left(\frac{\Lambda_{QCD}}{Q}\right)$$

PDF

$$x = k/p$$

$$f_1(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p|\bar{\psi}(0)\,\mathcal{L}(0,z^-)\,\frac{\gamma^+}{2}\,\psi(0,z^-,0_\perp)|P\rangle$$

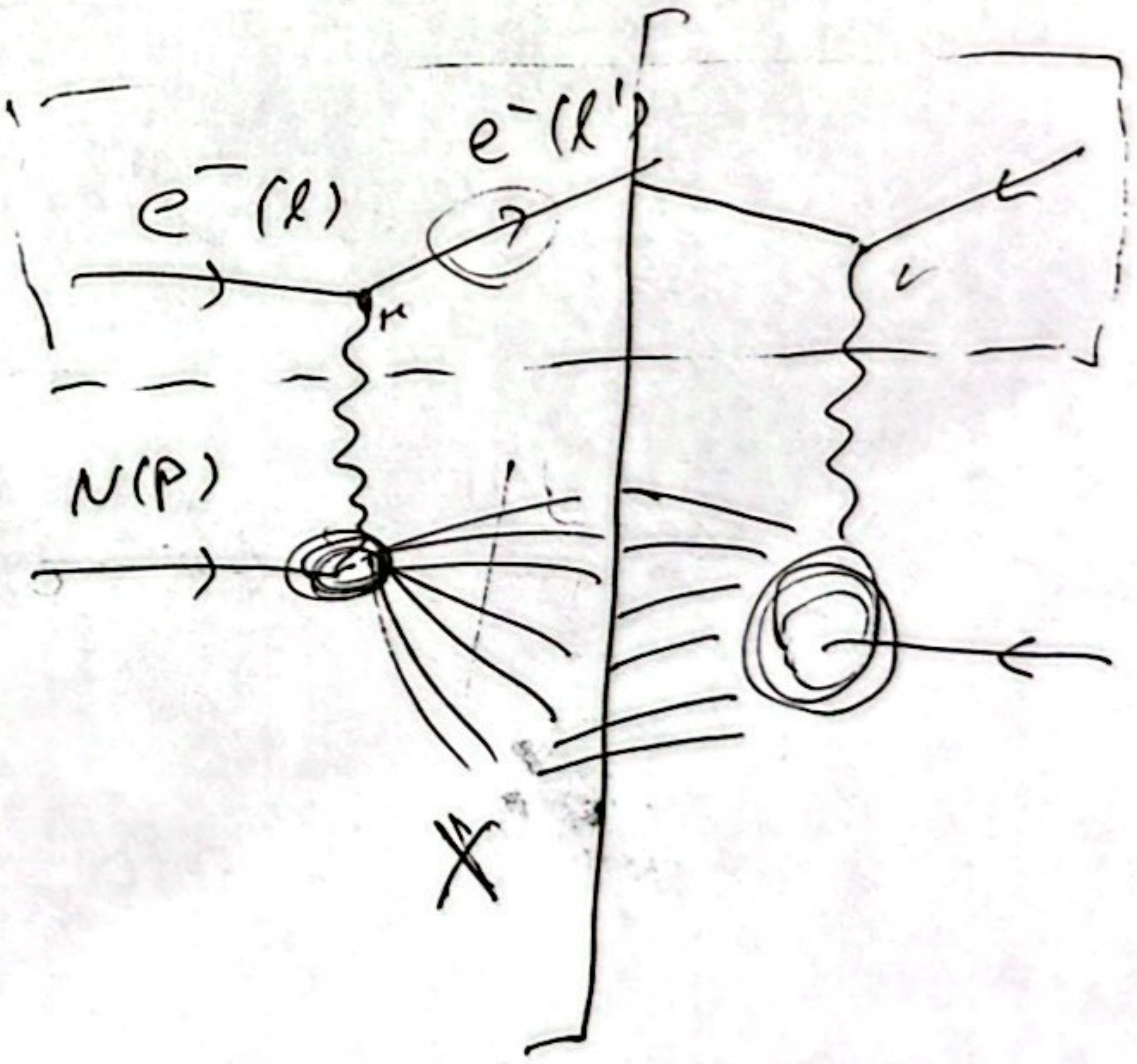
DIS

Inclusive deep inelastic scattering $\quad e^- p \to e^- X$

$$d\sigma = \frac{\alpha_{em}^2}{s\,Q^4} L^{\mu\nu}\, \underline{W_{\mu\nu}(q,p,S)}\, \frac{d^3 l'}{2E'}$$

$$Q^2 = -q^2, \quad x_B = \frac{Q^2}{2q\cdot p}$$

$$y = \frac{q\cdot p}{l\cdot p}$$



$$d\sigma = \frac{1}{4s}\,\frac{|M|^2}{TV}\,\frac{d^3\ell'}{(2\pi)^3 2E'} \qquad M = \langle f|\hat{S}|i\rangle = \langle e_f^- X|\hat{S}|e_i^- N\rangle$$

$$\hat{S} = T e^{i\int d^4x\, H_I(x)} = \cdots + \frac{i^2}{2}\int d^4x\, d^4y\, H_I(x) H_I(y)$$

$$H_I(x) = e J^\mu A_\mu = e\bar{\psi}\gamma_\mu\psi A_\mu$$

$$M = \frac{i^2}{2}\langle e_f^- X|\, T\int d^4x\, d^4y\, J_\mu(x)\underline{A^\mu(x)}\, J_\nu(y)\underline{A^\nu(y)}\,|e_i^- N\rangle$$

$$= i^2\int\frac{d^4q}{(2\pi)^4}\frac{-i}{q^2}\langle e_f^- X|\int d^4x\, d^4y\, e^{-iq(x-y)} J^\mu(x) J_\mu(y)|e_i^- N\rangle$$

$$= i^2\int\frac{d^4q}{(2\pi)^4}\frac{-i}{q^2}\int dx\, dy\, e^{iq(x-y)}\langle e_f^-|J^\mu(x)|e_i^-\rangle\langle X|J_\mu(y)|N\rangle$$

$$= i^2\int\frac{d^4q}{(2\pi)^4}\frac{-i}{q^2}\int d^4x\, d^4y\, e^{-i(\ell-\ell')x}\, e^{-i(P-P_X)y}\, e^{iq(x-y)}$$

$$\langle e_f^-|J^\mu(0)|e_i^-\rangle \quad \cancel{e^{ikx}} \quad \langle X|J_\nu(0)|N\rangle$$

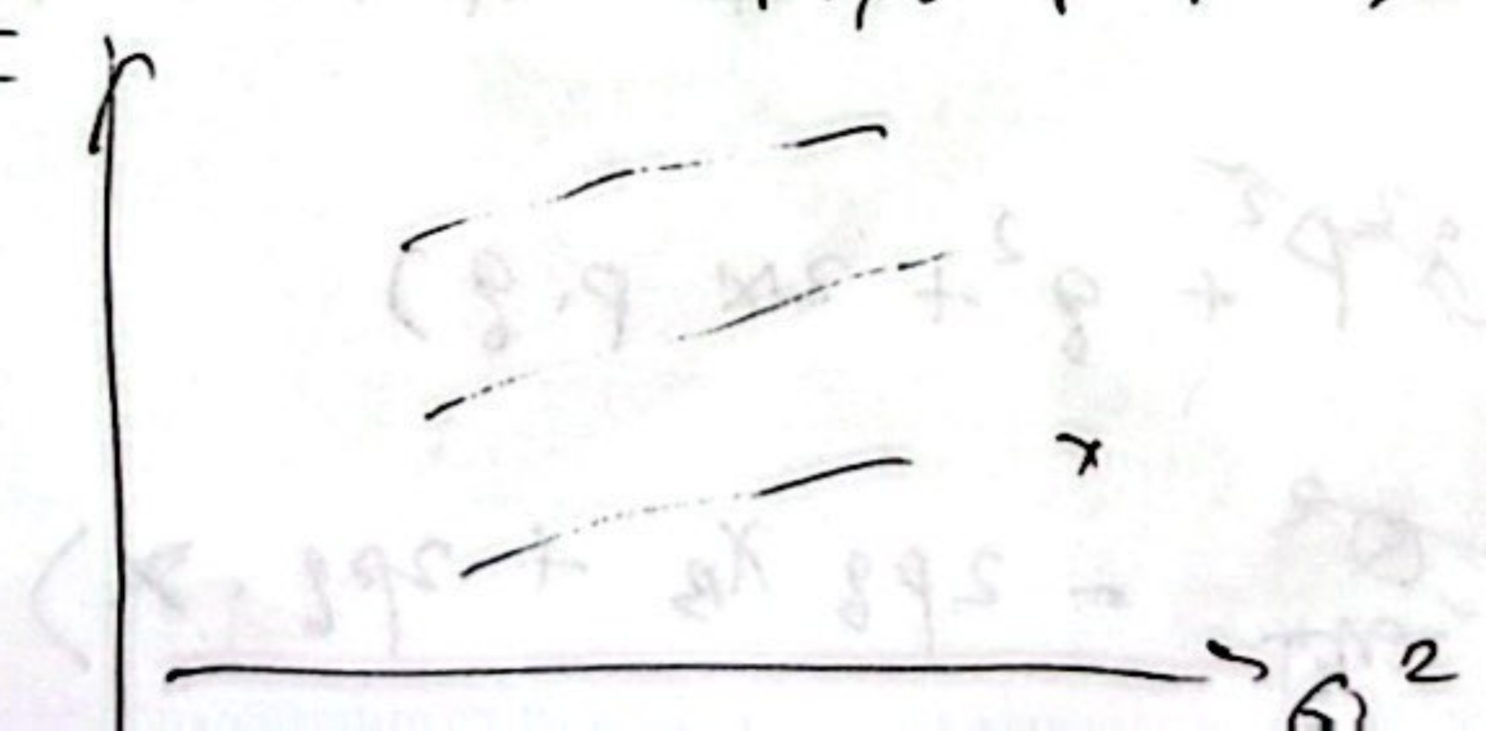
$$= \frac{i}{q^2}\langle e_f|J^\mu(0)|e_i^-\rangle\langle X|J_\mu(0)|N\rangle\,(2\pi)^4\delta^4(\ell+p-\ell'-P_X)$$

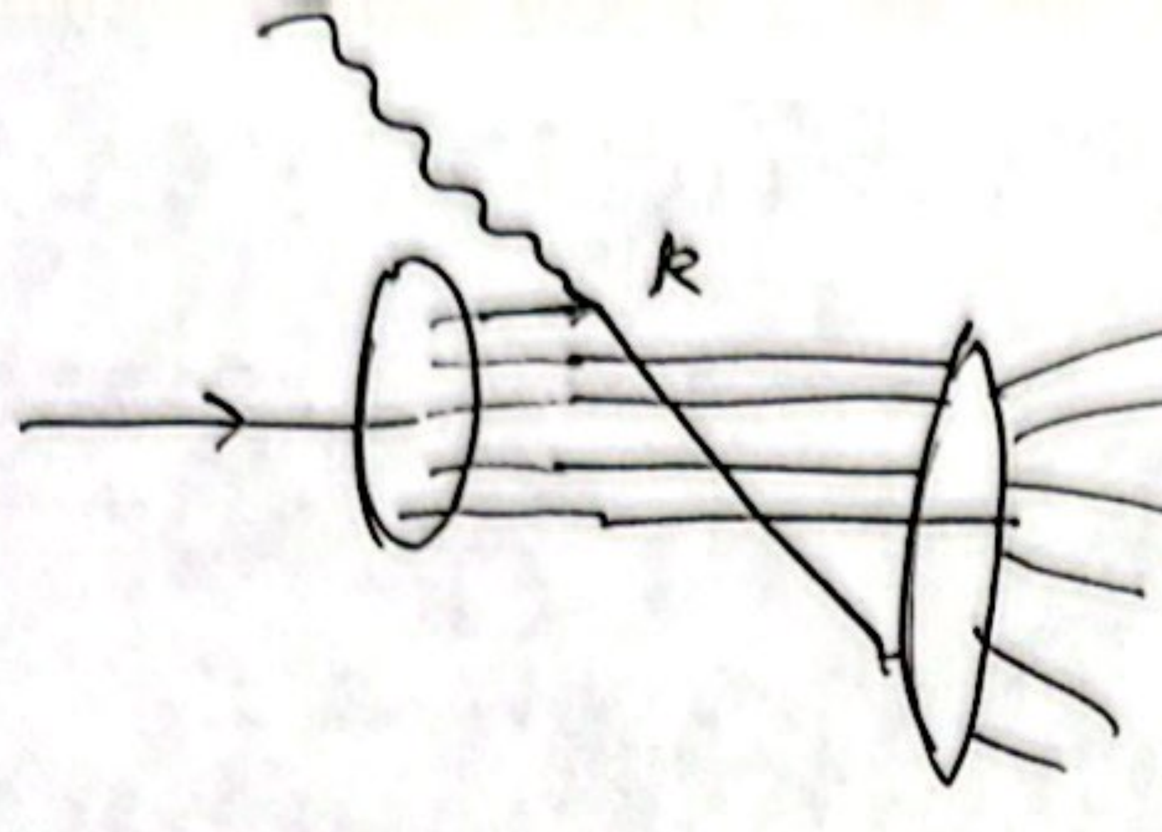
$$W_{\mu\nu}(q,P,S) = \sum_X \langle P,S|J_\mu(0)|X\rangle\langle X|J_\nu(0)|P,S\rangle\,(2\pi)^4\delta^4(p+q-P_X)$$

$$q^\mu W_{\mu\nu} = 0 \qquad W^*_{\mu\nu} = W_{\nu\mu}$$

$g^{\mu\nu}$ $\quad p^\mu p^\nu$ $\quad q^\mu q^\nu$ $\quad \epsilon_{\mu\nu\rho\sigma}p^\rho p^\sigma$ — —

$F_1 = x F_2$





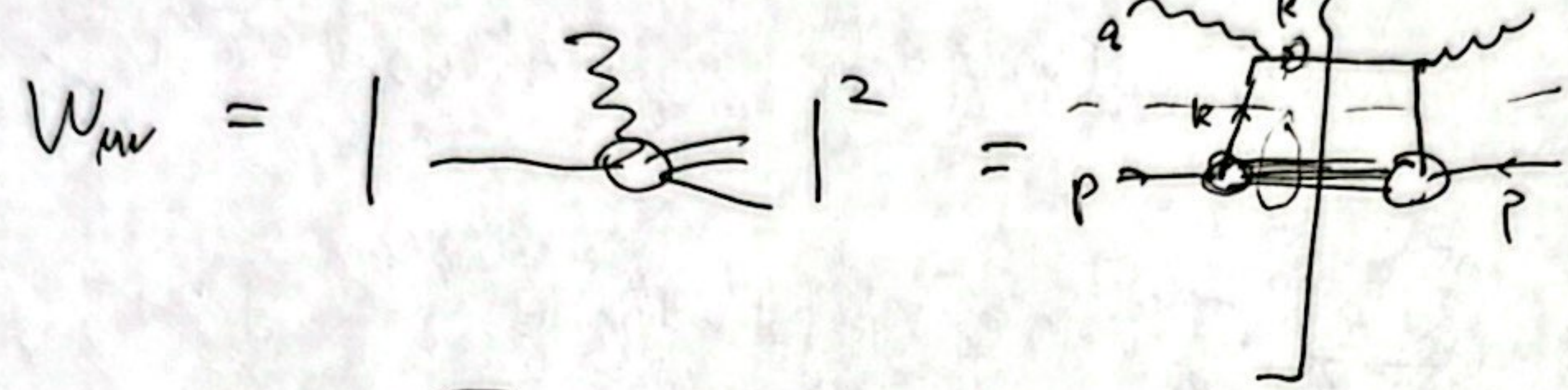
infinite momentum frame

$$P^+ = (P^0 + P^3)/\sqrt{2}$$

$$P^- = (P^0 - P^3)/\sqrt{2} \simeq 0$$

$$|M(eN \to eX)|^2 = \sum_q \int_0^1 dx \; f_q(x) \; |M(eq \to eq)|^2$$

$$x = k/P \qquad x_B \simeq x.$$



$$W_{\mu\nu}(q, p, S) = \sum_X \langle p, S| J^\mu(0) |X\rangle \langle X| J_\nu(0) |p, S\rangle (2\pi)^4 \delta^4(p + q - P_X)$$

$$= \sum_X \int d^4z \, \langle p, S| J_\mu(0) |X\rangle \langle X| J_\nu(z) |p, S\rangle e^{-iqz}$$

$$= \sum_{X'} \underline{\int \frac{d^4k'}{(2\pi)^4} \delta_+(k'^2)} \int d^4z \, e^{-iqz} \langle p, S| \bar\psi(0) |X'\rangle \underline{\gamma_\mu u(k') \bar u(k') \gamma_\nu} \; e^{ik'z} \langle X'| \psi(z) |p S\rangle$$

$$= \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat H_{\mu\nu}(k, q) \, \hat\phi(k, p, S) \right]$$

$$|X\rangle = |X'\rangle |k'\rangle$$

$$J_\mu = \bar\psi \gamma_\mu \psi$$

$$\psi(x) |X'\rangle |k'\rangle = u(k') e^{ik'x} |X'\rangle$$

$$\hat H_{\mu\nu}(k, q) = \gamma_\mu (\not k + \not q) \gamma_\nu (2\pi) \, \underline{\delta_+((k+q)^2)}$$

$$\phi(p, k) = \int d^4z \, e^{ikz} \langle p S| \bar\psi(0) \overset{\mathcal{L}(0, z^-)}{} \psi(z) |p S\rangle$$

Collinear approximation $\quad p = (p^+, 0, 0, 0), \quad k = x p.$

$$H_{\mu\nu} = \cdots \delta(\cancel{x^2 p^2} + q^2 + 2x \, p \cdot q) \qquad x_B = \frac{Q^2}{2pq}$$

$$= \delta\left(-\cancel{\frac{Q^2}{\;}} - 2pq \, x_B + 2pq \cdot x\right) \qquad x \simeq x_B$$

$$\hat{\phi}(x;p) = \int \frac{d^4k}{(2\pi)^4}\, \phi(k,p)\, \delta(x - k^+/p^+) \equiv \frac{1}{2} p^+ \bar{\not{n}}\, \underset{\sim}{f_1(x)}$$

$$= \frac{1}{(2\pi)^4} \int d^4k\, \delta(x - k^+/p^+) \int d^4z\, e^{ikz} \langle p | \bar{\psi}(0) \psi(z) | p \rangle$$

$$= \frac{1}{(2\pi)^4}\, p^+ \int \cancel{dz^+}\, dz^-\, \cancel{dz_\perp}\, e^{ixp_+ z^-} \langle p | \bar{\psi}(0) \psi(z) | p \rangle$$

$d^4k = dk^+ dk^- dk_\perp$

$k \cdot p = \underline{k^+ p^-} + k^- p^- - k_\perp \cdot p$

$$p^+ = p \cdot \bar{n} = \bar{\not{n}} \cdot \not{p} = \not{n}\, p^+ \gamma^+$$

$$\Rightarrow\ f_1(x) = \int \frac{dz^-}{(2\pi)}\, e^{ixp^+ z^-} \langle p | \bar{\psi}(0) \frac{\gamma^+}{2} \mathcal{L}(z^-, 0) \psi(z) | p \rangle$$