Chapter 8 Spontaneous symmetry breaking and Weinberg-Salam model

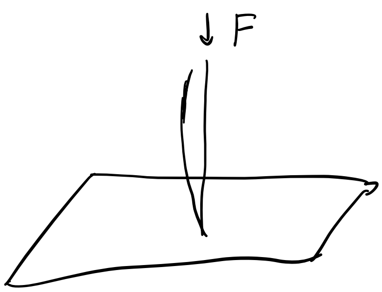
Nambu Goldstone

1964 Higgs

Weinberg Salam $SU(2)\times U(1)$

't Hooft

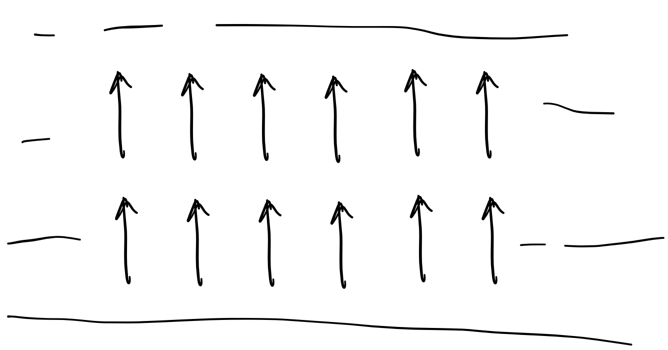
8.1 Vacuum?



(i) 某个参数具有临界值 (这里是F)
(ii) 对称的构型会不稳定
(iii) 基态简并

Ferromagnetism

$$H = -\sum_{ij} J_{ij}\, \vec{S_i}\cdot\vec{S_j}$$



高温下原子取向随机

参数：T

铁磁体无限大 $\xrightarrow{\text{类比}}$ 场论 ($f \to$ infinity)

Scalar field: ϕ^4 theory:

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - m^2 \phi^* \phi - \underbrace{\lambda (\phi^* \phi)^2}_{\text{self-interaction}}$$
$$= (\partial_\mu \phi)(\partial^\mu \phi^*) - V(\phi, \phi^*) \qquad (8.1)$$

$\mathcal{L}$ 相位变换下不变

$$\phi \to e^{i\Lambda} \phi \qquad (\Lambda\text{为常数}) \qquad (8.2)$$

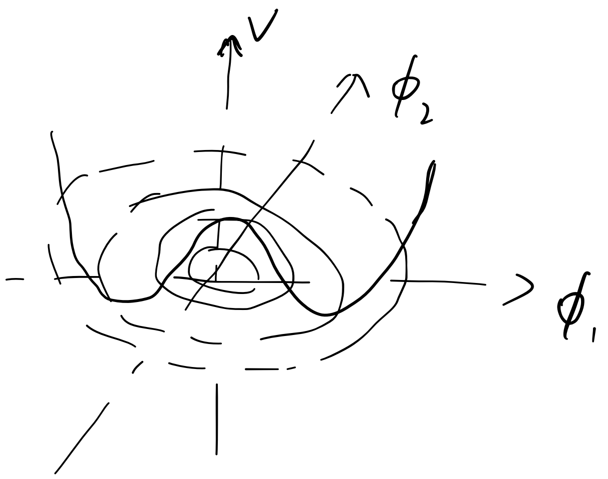
V 极小值 → 基态

$$\frac{\partial V}{\partial \phi} = m^2 \phi^* + 2\lambda \phi^* (\phi^* \phi) \qquad (8.3)$$

若 $m^2 > 0$，V_{min} 出现在 $\phi^* = \phi = 0$，若 $m^2 < 0$，会有局域极大值 ($\phi = 0$)，有极小值

$$|\phi|^2 = -\frac{m^2}{2\lambda} = a^2$$

$|\phi| = a$ Quantum theory: $|\langle 0|\phi|0\rangle|^2 = a^2$



$\phi(x) = \rho(x) e^{i\theta(x)}$

选择：$\langle 0|\phi|0\rangle = a$ 为真空态

$\Rightarrow \langle 0|\rho|0\rangle = a \qquad \langle 0|\theta|0\rangle = 0$

$$\phi(x) = [\rho'(x) + a] e^{i\theta(x)} \qquad (8.9)$$

ρ' 与 θ 真空态中都消失 “物理”场

$$V = m^2\rho'^2 + 2m^2 a\rho' + m^2a^2 + \lambda(\rho'^4 + 4a\rho'^3 + 6a^2\rho'^2 + 4a^3\rho' + a^4)$$
$$= \lambda(\phi^*\phi - a^2)^2 - \lambda a^4$$

$$(\partial_\mu\phi)(\partial^\mu\phi^*) = (\partial_\mu\rho')(\partial^\mu\rho') + (\rho'+a)(\partial_\mu\theta)(\partial^\mu\theta)$$

$\mathcal{L} = $ ——

ρ'^2 存在，被赋予质量

$m_{\rho'} = 4\lambda a^2$

无 θ^2 项 $\Rightarrow$ θ 为“无质量”场

θ，波长 λ，$\omega \to 0$，$\lambda \to \infty$ $\quad \omega \propto \frac{1}{\lambda}$，$E \propto p$ 在相对论下，无静质量

θ 粒子被称为 Goldstone 玻色子

Important：(连续)对称性自发破缺会导致存在无质量粒子，即 Goldstone 玻色子

$$\phi(x) = a + \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x)) \qquad (8.10)$$

$\langle\phi_1\rangle_0=0$, $\langle\phi_2\rangle_0=0$

$$\mathcal{L}=\frac{1}{2}(\partial_\mu\phi_1)^2+\frac{1}{2}(\partial_\mu\phi_2)^2-2\lambda a^2\phi_1^2-\sqrt{2}\lambda\phi_1(\phi_1^2+\phi_2^2)-\frac{\lambda}{4}(\phi_1^2+\phi_2^2)^2 \quad (8.11)$$

ϕ_2场无质量，ϕ_1场有 $m^2=4\lambda a^2$

"自旋波"

$w=ck$

若有长程力（如库仑力）/ 规范场

$w\to$ 有限 时，$\lambda\to\infty$，$k\to0$

光子也会这样

预测：无质量粒子（自旋为1）的规范粒子，自旋为0的Goldstone玻色子

§2 Goldstone定理：

Lag：U(1) 对称性

Q1：$\mathcal{L}$在对称群G下不变时，会有多少Goldstone玻色子？

Q2：这一切在量子理论中的status如何？特别是，如何证明给定简并真空态下存在无质量粒子？

SO(3)

$\phi_i\ (i=1,2,3)$ 是一个 Lorentz 标量的同位旋矢量场

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - \frac{m^2}{2}\phi_i\phi_i - \frac{\lambda}{4}(\phi_i\phi_i)^2 \qquad (8.12)$$

$\mathcal{L}$ 在同位旋空间下旋转不变. 旋转操作构成对称群 G:(SO(3))

$$G:\ \phi_i \to e^{iQ_k\alpha_k}\phi_i e^{-iQ_k\alpha_k} = (e^{-iT_k\alpha_k})_{ij}\phi_j = U_{ij}\phi_j = [U(g)\phi]_i \quad (8.13)$$

α_i 是同位旋空间下转动角, Q_i 为群生成元, T_i 是一组 遵循群的 Lie 代数的矩阵, 维度与群表示相同 (3维), $U(g)$ 为酉矩阵, 对应着群元 g (T 为厄米矩阵)

关注 V:

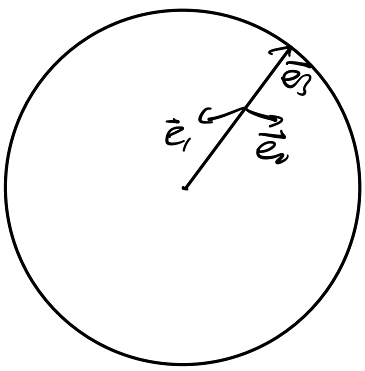
$$V = \frac{m^2}{2}\phi_i\phi_i + \lambda(\phi_i\phi_i)^2$$

$m^2>0$, $\phi_i=0$, $m^2<0$, 有

$$|\phi_0| = (\phi_1^2+\phi_2^2+\phi_3^2)^{1/2} = \left(-\frac{m^2}{4\lambda}\right)^{1/2} \equiv a$$

取 $\vec{\phi}_0 = a\vec{e}_3$ 作为真空态

ϕ_0 指向为同位旋空间中 3 的指向



ϕ_0 在 G 下不是不变量, 即 $g\in G$,

$$G:\ \phi_0' = U(g)\phi \neq \phi_0; \qquad (8.17)$$

但是在 G 的子群 H 中, ϕ_0 是旋转不变的. 即

$$H:\ \phi_0' = U(h)\phi_0 = \phi_0$$

$$U(h) = e^{iT_3\alpha_3}$$

在群G下，V是不变量

$$V(\phi')=V(\phi) \qquad \phi'=U(g)\phi$$

令 $\phi_3=\chi+a$

"物理场"：ϕ_1, ϕ_2, χ

$$V=\frac{m^2}{2}\left[\phi_1^2+\phi_2^2+(\chi+a)^2\right]+\lambda\left[\phi_1^2+\phi_2^2+(\chi+a)^2\right]^2$$
$$=\lambda\left[(\phi_i\phi_i-a^2)^2-a^4\right]$$

χ 具有二次项，故

$$m_\chi^2=8a^2\lambda \qquad m_{\phi_1}=m_{\phi_2}=0$$

推广：$V(\phi)$ 在极小值处展开

$$\frac{\partial V}{\partial \phi_a}\Big|_{\phi=\phi_0}=0$$

$$V=V(\phi_0)+\frac{1}{2}\left(\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\right)_{\phi=\phi_0}\chi_i\chi_j+O(\chi^3) \qquad (8.23)$$

其中 $\chi(x)=\phi(x)-\phi_0$

质量矩阵：

$$M_{ij}=\left(\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\right)_{\phi=\phi_0}\geq 0 \qquad (8.24)$$

$V(\phi_0)$ 最小

作群变换

$$V(\phi_0)=V(U(g)\phi_0)=V(\phi_0)+\frac{1}{2}\left(\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\right)_{\phi_0}\delta\phi_i\delta\phi_j+\cdots$$

因此，

$$\left(\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\right)\delta\phi_i\delta\phi_j=0$$

$\delta\phi_i$ 为 ϕ 变换后的变分

g 是否属于 H：

是：$\phi_0' = \phi_0$，$\delta\phi_i = 0$

$$\delta\phi_i = \left(\frac{\partial u}{\partial \alpha_3}\right)_{\alpha_3=0} \phi_0 \delta\alpha_3 = 0 \qquad (8.26)$$

否：$\delta\phi_m = \left[\left(\frac{\partial u}{\partial \alpha_i}\right)_{\alpha_i=0} \phi_0\right]_m \delta\alpha_i \neq 0 \qquad (8.27)$

$$M_{ij}[U'(0)\phi_0]_j = 0$$

$U'(0)\phi_0$ 不是为 0. 这些都是被 Goldstone 的.

$H = SO(2) \sim U(1)$ （生成元 T_3）

G/H

适用对称性不会自发破缺的情况.

$H = G$

$G/H = G$

$\phi(x)$ 具有 $\langle 0|\phi(x)|0\rangle \neq 0$

$$j_\mu^a(x) = \frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi)} \frac{\delta\phi(x)}{\delta\alpha^a}$$

无散度 $\Rightarrow \partial^\mu j_\mu^a = 0$

$$Q^a = \int d^3x\, j_0^a(x) \qquad (8.28)$$

守恒.

$\frac{dQ^a}{dt} = 0$

$[Q^a, Q^b] = \underset{\sim}{C^{abc}} Q^c$

$U = e^{iQ^a \alpha^a}$ (8.29)

若 $U|0\rangle = |0\rangle$，那么

$\underline{Q^a |0\rangle = 0}$

若无此情况，有：简并真空态：

$U|0\rangle = |0\rangle' = |0\rangle$ or $Q^a|0\rangle \neq 0$

不是本征态，从

$\phi(x)$ 有 $\phi'(x)$

$[Q^a, \phi'(x)] = \phi(x)$

$\langle 0|\phi(x)|0\rangle \neq 0$

$\underline{\langle 0|[Q^a, \phi'(x)]|0\rangle \neq 0}$ (8.32)

$$\sum_n \int d^3y \left[\langle 0|j_0^a(y)|n\rangle\langle n|\phi'(x)|0\rangle - \langle 0|\phi'(x)|n\rangle\langle n|j_0^a(y)|0\rangle\right]\Big|_{x^0=y^0} \neq 0 \quad (8.33)$$

$$j_0^a(y) = e^{iPy} j_0^a(0) e^{iPy}$$

$$\sum_n \int d^3y \left[\langle 0|j_0^a(y)|n\rangle\langle n|\phi'(x)|0\rangle - \langle 0|\phi'(x)|n\rangle\langle n|j_0^a(y)|0\rangle\right]\Big|_{x^0=y^0} \neq 0$$

$$= (2\pi)^3 \sum_n \delta^3(\vec{p}_n)\left[\langle 0|j_0^a(0)|n\rangle\langle n|\phi'(x)|0\rangle e^{iP_{n0}y_0} - \langle 0|\phi'(x)|n\rangle\langle n|j_0^a(0)|0\rangle e^{-iP_{n0}y_0}\right]\Big|_{x^0=y^0}$$

$$= (2\pi)^3 \sum_n \delta^3(\vec{p}_n)\left[\langle 0|j_0^a(0)|n\rangle\langle n|\phi'(x)|0\rangle e^{iM_n y_0} - \langle 0|\phi'(x)|n\rangle\langle n|j_0^a(0)|0\rangle e^{-iM_n y_0}\right]\Big|_{x^0=y^0}$$

$\neq 0$ (8.34)

$(|n\rangle = |0\rangle)$

$$\partial^\mu j_\mu^a(y) = \partial_0 j_0^\mu(y) + \nabla \cdot \vec{j}^a(y) = 0$$

$$\frac{\partial}{\partial y_0}\int d^3y \, j_0^a(y) = -\int d^3y \, \nabla\cdot\vec{j}^a(y)$$

$$\frac{\partial}{\partial y_0}\langle 0|[Q^a,\phi(x)]|0\rangle = \frac{\partial}{\partial y_0}\int d^3y \, \langle 0|[j_0^a(y),\phi(x)]|0\rangle$$

$$= -\int d^3y \, \langle 0|[\nabla\cdot\vec{j}^a(y),\phi(x)]|0\rangle$$

$$= -\int d\vec{S}\cdot\langle 0|[\vec{j}^a(y),\phi(x)]|0\rangle$$

(Guralnik, Hagen & Kibble in Cool & Marshak 1968)

partially conserved axial current