Electro-Weak Theory

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$SU(2)_L \times U(1)_Y$ symmetry.
neutral current ...?

1. $SU(2)_L \times U(1)_Y$ symmetry.

leptons: $\begin{pmatrix} e \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$ quarks: $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$

$SU(2)$ → $SU(2)_L$ doublet $L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ $Q = \begin{pmatrix} c \\ s \end{pmatrix}_L$ isospin: 同位旋.
→ $SU(2)_R$ singlet e_R μ_R u_R d_R

$$\mathcal{L}_{\text{Fermions}} = \underbrace{\bar{L} i \not{\partial} L + \bar{Q} i \not{\partial} Q}_{SU(2)_L} + \underbrace{\bar{e}_R i \not{\partial} e_R + \bar{\mu}_R i \not{\partial} \mu_R + \cdots}_{SU(2)_R}$$ Gauge problem!

gauge transformation: $\psi_L = L/Q$. ψ_L from $\frac{1}{2}(1-\gamma_5)\psi$.

$$\begin{cases} \psi_L \to e^{i\alpha^i \tau^i/2} e^{i\beta Y_L/2} \psi_L \\ \quad\;\; e^{i[\alpha^i\tau^i/2 + \beta Y_L/2]}\psi_L \\ \psi_R \to e^{i\beta Y_R/2}\psi_R \end{cases}$$

$e^X e^Y \neq e^{X+Y}$ (Here is OK!).

Y_L, Y_R: hypercharge (number)

$Y_L = Y_R = Y$: 盖尔曼-西岛关系: $Y/2 = Q - t_3$.

for ψ_L: $t_3 = \begin{cases} \frac{1}{2} & \nu.\ u.c.t \\ -\frac{1}{2} & e.\mu.\tau.d.s.b. \end{cases}$ messy!

Is $U(1)_Y$ same as $U(1)_Q$? The answer is not.

Q: → doublet : $\begin{cases} \begin{pmatrix} 0 & \\ & -1 \end{pmatrix} & \text{for leptons} \\ \begin{pmatrix} \frac{2}{3} & \\ & -\frac{1}{3} \end{pmatrix} & \text{for quarks} \end{cases}$ $SU(2)_L$: generators are $\tau^i/2$

they don't commute? ⇒ $U(1)_Q \times SU(2)_L$ is bad!

$$\mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Fermions}} = -\frac{1}{4}F^{i\mu\nu}F^i_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + \mathcal{L}'_{\text{Fermions}}.$$ prime means $\not{\partial} \to \not{D}$.

$$\Rightarrow \mathcal{L}'_{\text{Fermions}} = \bar{e}_R i \not{D}_R e_R + \bar{L} i \not{D}_L L$$

interactions

$$\begin{cases} D^\mu_L = \partial^\mu - ig\frac{\tau^i}{2}W^{i\mu} - ig'\frac{Y_L}{2}B^\mu \\ D^\mu_R = \partial^\mu - ig'\frac{Y_R}{2}B^\mu. \end{cases}$$

$W^{i\mu}$ for $F^{i\mu\nu}$
B^μ for $B^{\mu\nu}$

Define: $W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)$ $W^3_\mu \to A_\mu$. $B_\mu \to Z_\mu$.

↓ charged weak intermediate bosons ↓ photon ↓ special field: coupling to weak neutral currents.

How to get their masses? SSB: Higgs Mechanism.

$SU(2)_L \times U(1)_Y \to U(1)_Q$

$$\mathcal{L} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Fermions}} + \mathcal{L}_S$$

$\mathcal{L}_S = D_\mu\phi^\dagger D^\mu\phi - V(\phi)$. ϕ: should be $SU(2)_L$ doublet.

$\Rightarrow D_\mu = (\partial_\mu - ig\frac{\tau_i}{2}W^i_\mu - ig'\frac{Y_\phi}{2}B_\mu)$ $\phi \to e^{i\alpha^i\tau^i/2} e^{i\beta/2}\phi$.

$\beta(x)$. $\alpha^i(x)$ → coordinate dependent.

SSB calculation: (1). Vaccum expectation value: $\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$

(2) expand near the vacuum:

$$\Delta\mathcal{L} = \frac{1}{2}\boxed{\frac{v^2}{4}[g^2}(W^1_\mu)^2 + g^2(W^2_\mu)^2 + (-gW^3_\mu + g'B_\mu)^2]$$

recall that: $W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)$ ⇒ $m_W = \frac{gv}{2}$ (W boson)

$g. g'$: $\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$ $\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$ θ_W: Weinberg angle

$$\left.\begin{aligned} B^\mu &= \cos\theta_W A^\mu - \sin\theta_W Z^\mu \\ W^\mu_3 &= \sin\theta_W A^\mu + \cos\theta_W Z^\mu \end{aligned}\right\} \sim \frac{g^2}{\cos^2\theta_W}Z_\mu Z^\mu \Rightarrow m_Z = \frac{m_W}{\cos\theta_W}$$ (Z boson).

from 4-fermions theory : $G_F \approx 1.166 \times 10^{-5}$ GeV^{-2} $\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$ ⇒ $v \approx 246$ GeV.

from experiment : $\sin^2\theta_W \approx 0.23$ ⇒ $\begin{cases} m_W \approx 80\,\text{GeV} \\ m_Z \simeq 90\,\text{GeV} \end{cases}$ $m_A = 0$ massless.

Why EM-field? $\tau^i\begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0$ $Y\begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0$

$Q = (\frac{Y}{2} + \frac{\tau^3}{2}) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ ($Y=1$) $Q\begin{pmatrix} 0 \\ v \end{pmatrix} = 0$. EM charge satisfy $U(1)_Q$ symmetry.

Gauge sector. Higgs sector. Fermions sector