Spontaneous symmetry breaking for gauge fields

$$\underline{\phi(x) \to e^{ie\Lambda(x)}\phi(x)}$$

$$\mathcal{L} = (\partial_\mu + ieA_\mu)\phi(\partial^\mu - ieA^\mu)\phi^* - m^2\phi^*\phi - \lambda(\phi^*\phi)^2 - \frac14 F_{\mu\nu}F^{\mu\nu}$$

m^2 参数. $m^2 < 0$

$$|\phi| = a = \left(\frac{-m^2}{2\lambda}\right)^{1/2}$$

$$\phi(x) = a + \frac{\phi_1(x) + i\phi_2(x)}{\sqrt{2}}$$

$$\mathcal{L} = -\frac14 F_{\mu\nu}F^{\mu\nu} + \underbrace{e^2a^2A_\mu A^\mu}_{\text{photon}} + \frac12(\partial_\mu\phi_1)^2 + \frac12(\partial_\mu\phi_2)^2 - 2\lambda a^2\phi_1^2 + \underline{\sqrt{2}eaA^\mu\partial_\mu\phi_2} + \text{三次项} + \text{四次项}$$

ϕ_1 质量场 ϕ_2 无质量场?

$$\begin{cases} \phi_1' = \phi_1 - \Lambda\phi_2 \\ \phi_2' = \phi_2 + \Lambda\phi_1 + \sqrt{2}\Lambda a \end{cases}$$

选择$\Lambda \to \phi_2 = 0$

$$\mathcal{L} = -\frac14 F_{\mu\nu}F^{\mu\nu} + e^2a^2A_\mu A^\mu + \frac12(\partial_\mu\phi_1)^2 - 2\lambda a^2\phi_1^2 + \text{耦合项}$$

Goldstone 模式 (全局U(1) 对称性 SB)

2个scalar 质量场 $\to$ 1个 scalar 质量场 + 1个无质量 标量场

Higgs 模式

2个 scalar 标量场 + 1个 photon $\to$ 1 scalar 质量场 + 1个有质量的 photon

Gupta - Bleuler 机制

O(3)

$$\mathcal{L} = \frac{1}{2}(D_\mu\phi_i)(D^\mu\phi_i) - \frac{m^2}{2}\phi_i\phi_i - \lambda(\phi_i\phi_i) - \frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu}$$

$$\begin{cases} D_\mu\phi_i = \partial_\mu\phi_i + g\varepsilon_{ijk}A^j_\mu\phi_k \\ F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\varepsilon^{ijk}A^j_\mu A^k_\nu \end{cases}$$

$m^2 < 0 \Rightarrow V_{min} \Rightarrow \quad |\phi_0| = \left(-\frac{m^2}{4\lambda}\right)^{1/2} = a$

$$\vec{\phi}_0 = a\hat{e}_3$$

$\phi_1, \phi_2, \chi = \phi_3 - a$

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu\phi_1)^2 + (\partial_\mu\phi_2)^2 + (\partial_\mu\chi)^2] + ag[(\partial_\mu\phi_1)A_2^\mu - (\partial_\mu\phi_2)A_1^\mu]$$

$$+ \frac{a^2g^2}{2}[(A^1_\mu)^2 + (A^2_\mu)^2] - \frac{1}{4}(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)^2 - 4a^2\lambda\chi^2 + 三次項 + 四次項$$

$$\vec{\phi}(x) = \hat{e}_3\phi_3 = \hat{e}_3(a+\chi)$$

$$D_\mu\phi_1 = g(a+\chi)A^2_\mu$$

$$D_\mu\phi_2 = -g(a+\chi)A^1_\mu$$

$$D_\mu\phi_3 = \partial_\mu\chi$$

$$(D_\mu\phi_i)^2 = a^2g^2[(A^1_\mu)^2 + (A^2_\mu)^2] + (\partial_\mu\chi)^2 + \cdots$$

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu^i - \partial_\nu A_\mu^i)^2 - \frac{1}{2}a^2 g^2\left[(A_\mu^1)^2 + (A_\mu^2)^2\right] + \frac{1}{2}(\partial_\mu \chi)^2 - 4a^2\lambda\chi^2 + 三次项 + 四次项$$

$O(3)$:

Goldstone 模式（全局 $O(3)$ 对称性 SB）

3个scalar 质量场 $\longrightarrow$ 1个 scalar 质量场 + 2个无质量 标量场

Higgs 模式（局部 $O(3)$ 对称性 SB）

3个 scalar 标量场 + 1个 photon $\longrightarrow$ 1个scalar质量场 + 2个 vector 质量场 + 1个 vector 无质量场

Superconductivity:

$\partial_0 \phi = 0$

$$\mathcal{L} = -(\nabla - ie\vec{A})\phi \cdot (\nabla + ie\vec{A})\phi^* - m^2|\phi|^2 - \lambda|\phi|^4 - \frac{1}{2}(\nabla\times\vec{A})^2$$

$$-\mathcal{L} = \frac{1}{2}(\nabla\times\vec{A}) + |(\nabla - ie\vec{A})\phi|^2 + m^2|\phi|^2 + \lambda|\phi|^4$$

$m^2 = a(T - T_c)$

Bardeen–Cooper–Schrieffer

$T > T_c$ 时，$|\phi| = 0$ 处 自由能最小

$T < T_c$ $\quad m^2 < 0$

$$|\phi|^2 = -\frac{m^2}{2\lambda} > 0$$

$\mathcal{L}$ 在相位变换 $\phi \to e^{i\Lambda(x)}\phi$，$\vec{A} \to \vec{A} + \frac{1}{e}\nabla\Lambda(x)$

$$\vec{j} = -i(\phi^*\nabla\phi - \phi\nabla\phi^*) - 2e|\phi|^2\vec{A}$$

$$\vec{j} = \frac{em^2}{\lambda}\vec{A} = -k^2 A$$

$$\begin{cases} \vec{E} = -\dfrac{\partial \vec{A}}{\partial t} = 0 \\ \vec{E} = R\vec{j} \end{cases} \Rightarrow R = 0$$

$$\nabla \times \vec{B} = \vec{j}$$

$\nabla^2 B = k^2 B$ (设定一维) $B_x = B_0 e^{-kx}$

$\Rightarrow \nabla^2 \vec{A} = k^2 \vec{A}$

$1/k \approx 10^{-6}\ \mathrm{cm}$ $\qquad \Box A_\mu = -k^2 A_\mu$