

Pion-nucleon scattering amplitude

$$S = : \exp \left[ \int (\bar{\psi} i \not{\partial} \psi + F_{,n} \bar{D} \frac{\delta}{\delta \eta} - \frac{\delta}{\delta \bar{\eta}} \bar{D} \psi_n) dx \right] : Z[J, \eta, \bar{\eta}] |_0$$

其  $D = i \gamma^\mu \partial_\mu - M$      $\bar{D} = -i \overleftarrow{\gamma}^\mu \partial_\mu - M$

$$L_{int} = i g \bar{\psi} \gamma_5 \vec{\tau} \psi \cdot \vec{\phi}$$

定  $\alpha$

$$\phi^\mp = \frac{1}{\sqrt{2}} (\phi_1 \pm i \phi_2), \quad \phi^0 = \phi_3$$

$$L_{int} = i \sqrt{2} g (\bar{p} \gamma_5 n \phi^+ + \bar{n} \gamma_5 p \phi^-) + i g (\bar{p} \gamma_5 p - \bar{n} \gamma_5 n) \phi^0$$

$\pi^+ p$

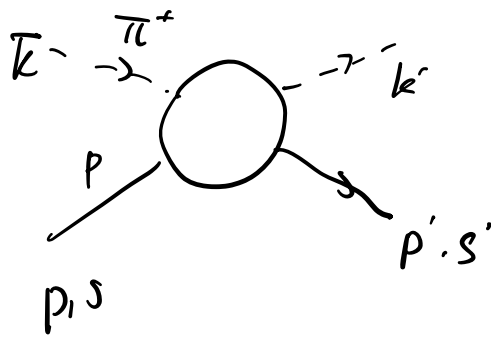
$$L_{int} = i \sqrt{2} g (\bar{p} \gamma_5 n \phi^+ + \bar{n} \gamma_5 p \phi^-)$$

$$L = L_0 + L_{int} = \bar{\psi} \gamma^\mu \partial_\mu \psi + M \bar{\psi} \psi + \frac{1}{2} \partial^\mu \vec{\phi} \partial_\mu \vec{\phi} - \frac{1}{2} m^2 \vec{\phi}^2 + i \sqrt{2} g (\bar{p} \gamma_5 n \phi^+ + \bar{n} \gamma_5 p \phi^-)$$

$$Z[J, \eta, \bar{\eta}] = \frac{\exp \left\{ i \int \left( i \sqrt{2} g \frac{\delta}{\delta \eta(x)} \gamma_5 \frac{\delta}{\delta \bar{\eta}(y)} - \frac{\delta}{\delta J(x)} \right) dx \right\} Z_0[J, \eta, \bar{\eta}]}{[\dots] |_{J=\eta=\bar{\eta}=0}}$$

$$= \frac{\int \mathcal{D}\psi \mathcal{D}\eta \mathcal{D}\bar{\eta} \exp \left[ i \int (\bar{\psi} \not{\partial} \psi - \frac{1}{2} \phi(x) \square \phi(x) + \bar{\eta}(x) \psi(x) + \bar{\psi}(x) \eta(x)) dx \right]}{[\dots] |_{J=\eta=\bar{\eta}=0}}$$

$$= \exp \left\{ -i \int \left[ \frac{1}{2} J(x) \Delta_F(x-y) J(y) + \bar{\eta}(x) S(x-y) \eta(y) \right] dx dy \right\}$$



$$S_{fi} = \langle p', s'; k' | S | p, s; k \rangle$$

$$S = : \exp \left[ \int d^4x \left( \phi_{in} \overleftrightarrow{\partial} \frac{\delta}{\delta J} + \bar{\psi}_{in} \overleftrightarrow{\partial} \frac{\delta}{\delta \eta} - \frac{\delta}{\delta \eta} \overleftarrow{\partial} \psi_{in} \right) \right] : Z[J, \eta, \bar{\eta}] / Z_0$$

$$|p, s; k\rangle = b_s^\dagger(p) a^\dagger(k) |0\rangle$$

$$\langle p', s'; k | = \langle 0 | b_s(p') a(k')$$

$$S_{fi} = \langle 0 | b_s(p') a(k') \left[ 1 + \int d^4x \left( \phi_{in} \overleftrightarrow{\partial} \frac{\delta}{\delta J} + \bar{\psi}_{in} \overleftrightarrow{\partial} \frac{\delta}{\delta \eta} - \frac{\delta}{\delta \eta} \overleftarrow{\partial} \psi_{in} \right) + \dots \right] Z \Big| \times b_s^\dagger(p) a^\dagger(k) |0\rangle$$

$$b_s(p) |0\rangle = a(k) |0\rangle$$

$$\alpha_i \lambda \phi_{in} \quad \phi$$

$$\begin{aligned} \langle 0 | a(k') \int d^4x \frac{d^4q}{[(2\pi)^4 2uq]^{1/2}} [f_1(x) a(q) + f_2(x) a^\dagger(q)] a^\dagger(k) |0\rangle \overleftrightarrow{\partial} \frac{\delta}{\delta J} \\ = \int \langle 0 | a(k') \int dx \frac{d^4q}{[(2\pi)^4 2uq]^{1/2}} f_1(x) [a(q), a^\dagger(k)] |0\rangle \\ + \langle 0 | \int dx \frac{d^4q}{[(2\pi)^4 2uq]^{1/2}} f_2(x) [a(k), a^\dagger(q)] a^\dagger(k) |0\rangle \Big| \overleftrightarrow{\partial} \frac{\delta}{\delta J} \\ = 0 \end{aligned}$$

$$\frac{1}{2} \left( \overleftrightarrow{\partial} \left[ \frac{\delta}{\delta J} \right] \right)^2$$

$$\langle 0 | a(k') \int dx_1 dx_2 \frac{d^4q}{[(2\pi)^4 2uq]^{1/2}} \frac{d^4q'}{[(2\pi)^4 2uq']^{1/2}} [f_1(x_1) f_2(x_2) a(q) a(q') + f_2(x_1) f_1(x_2) a^\dagger(q) a(q') + f_1(x_1) f_2(x_2) a^\dagger(q) a(q')] \Big|$$

$$+ f_{q'}^*(x_1) f_{q'}^*(x_2) a^\dagger(q) a^\dagger(q')] a^\dagger(k) |0\rangle$$

$$\begin{aligned} \langle 0 | a(k') a^\dagger(q) a(q) a^\dagger(k) |0\rangle &= \langle 0 | [a(k'), a^\dagger(q)] [a(q), a^\dagger(k)] |0\rangle \\ &= (2\pi)^6 4\omega_{k'} \omega_k \delta^3(k'-q) \delta^3(q-k) \end{aligned}$$

$$\langle 0 | a(k') a^\dagger(q) a(q') a^\dagger(k) |0\rangle = (2\pi)^6 4\omega_{k'} \omega_k \delta^3(k'-q) \delta^3(k-q')$$

$$\begin{aligned} &\int dx_1 dx_2 \frac{d^3q d^3q'}{(4\omega_q \omega_{q'})^2} 4\omega_k \omega_{k'} \frac{1}{(4\omega_q \omega_{q'})} [e^{-i(qx_1 - q'x_2)} \times \delta^3(k'-q) \delta^3(q-k) + e^{i(qx_1 - q'x_2)} \\ &\quad \delta^3(k'-q) \delta^3(k-q')] \\ &= \int dx_1 dx_2 (e^{-ikx_1} e^{ik'x_2} + e^{ik'x_1} e^{-ikx_2}) \end{aligned}$$

$$\frac{1}{2} \left[ K_{x_1} \frac{\delta}{\delta J(x_1)} K_{x_2} \frac{\delta}{\delta J(x_2)} Z[J, \bar{\eta}, \eta] \right] \Big|_{J=\eta=\bar{\eta}=0} \quad \text{ZS}$$

$$\int dx_1 dx_2 e^{-ikx_1} \frac{1}{2} K_{x_2} \frac{\delta}{\delta J(x_1)} \frac{\delta}{\delta J(x_2)} Z[J, \bar{\eta}, \eta] \Big|_{J=\eta=\bar{\eta}=0} \leftarrow K_{x_1} e^{-ikx_1}$$

$$\int dx_3 dx_4 \langle 0 | b_s(p') : \left[ \bar{\Psi}(x_4) D_{x_4} \frac{\delta}{\delta \eta(x_4)} - \Psi(x_4) \frac{\delta}{\delta \eta(x_4)} \bar{D}_{x_4} \right]$$

$$\times \left[ \bar{\Psi}(x_3) D_{x_3} \frac{\delta}{\delta \eta(x_3)} - \Psi(x_3) \frac{\delta}{\delta \eta(x_3)} \bar{D}_{x_3} \right] b_s^\dagger(p) |0\rangle$$

$$: \bar{\Psi}(x_4) \bar{\Psi}(x_3) : \quad : \Psi(x_4) \Psi(x_3) : \quad \text{vanish}$$

$$: \bar{\Psi}(x_4) \Psi(x_3) :$$

$$= \langle 0 | b_s(p') \int \frac{d^3q d^3q'}{(2\pi)^6} \frac{M^2}{q_0 q_0'} \sum_{\alpha} : \left[ b_2^\dagger(q) \bar{u}^\alpha(q) e^{iqx_4} + d_2(q) \bar{v}^\alpha(q) e^{-iqx_4} \right.$$

$$\left. \times \left[ b_2(q') u^\alpha(q') e^{-iq'x_3} + d_2^\dagger(q') v^\alpha(q') e^{iq'x_3} \right] : b_s^\dagger(p) |0\rangle$$

$$= - \int \frac{d^3q d^3q'}{(2\pi)^6} \frac{M^2}{q_0 q_0'} \sum_{\alpha} \langle 0 | b_s(p') b_2^\dagger(q) b_2(q') b_s^\dagger(p) |0\rangle \times \bar{u}^\alpha(q) u^\alpha(q') e^{iqx_4} e^{-iq'x_3}$$

$$= - \bar{u}^s(p') u^s(p) e^{ip'x_4} e^{-ipx_3}$$

$$: \psi(x_4) \bar{\psi}(x_3) :$$

$$\frac{1}{2} D_{x_4} (\delta / \delta \bar{\eta}(x_4)) (\delta / \delta \eta(x_3)) \bar{D}_{x_3} \quad \frac{1}{2} (\delta / \delta \eta(x_4)) \bar{D}_{x_4} D_{x_3} (\delta / \delta \bar{\eta}(x_3))$$

$$\delta / \delta \eta, \delta / \delta \bar{\eta}$$

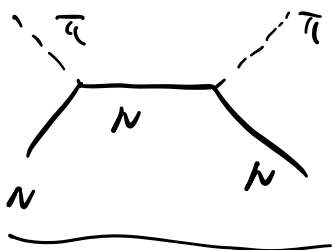
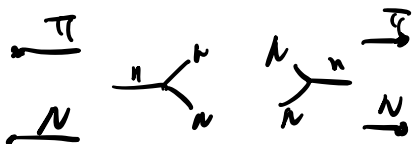
$$\int dx_3 dx_4 e^{i p' x_4} \bar{u}^s(p') D_{x_4} \frac{\delta}{\delta \eta(x_3)} \frac{\delta}{\delta \bar{\eta}(x_4)} Z[J, \eta, \bar{\eta}] \Big|_{J=\eta=\bar{\eta}=0} \bar{D}_{x_3} u^s(p) e^{-i p x_3}$$

$$S_{fi} = \int dx_1 \dots dx_4 e^{i k' x_2} e^{i l' x_4} \bar{u}^s(p') \overrightarrow{K}_{x_2} \overrightarrow{D}_{x_4} \underbrace{T(x_1, \dots, x_4)}_{\overleftarrow{D}_{x_2} \overleftarrow{K}_{x_1}} u^s(p) e^{-i p x_3} e^{-i k x_1}$$

$$T(x_1, x_2, x_3, x_4) = \frac{\delta}{\delta J(x_1)} \frac{\delta}{\delta \bar{J}(x_2)} \frac{\delta}{\delta \eta(x_3)} \frac{\delta}{\delta \bar{\eta}(x_4)} Z[J, \eta, \bar{\eta}] \Big|_{J=\eta=\bar{\eta}=0}$$

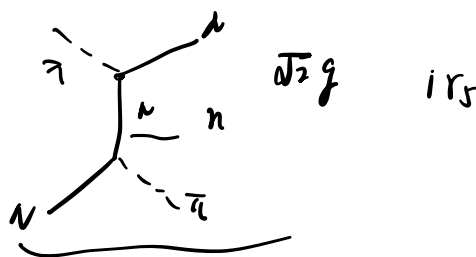
$$\phi \psi \quad i \psi \gamma_5 \psi$$

$$(L(N)) \rightarrow (T(N))$$

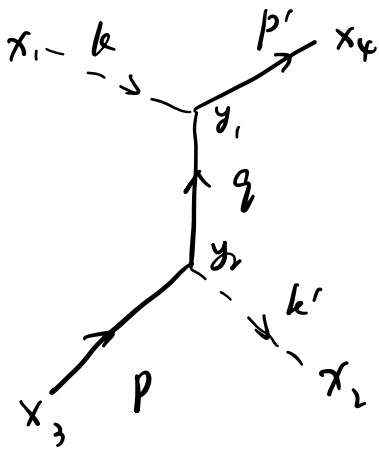


$$\langle \phi_{out} | S | \phi_{in} \rangle$$

$$N^{++}$$



$$T(x_1, x_2, x_3, x_4) = 2g^2 \int dy_1 dy_2 i \Delta_F(x_2 - y_2) i S(x_4 - y_1) i \gamma_5 \times i S(y_1 - y_3) i \gamma_5 i S(y_3 - x_3) i \Delta_F(x_1 - y_1)$$



$$\begin{cases} K_x \Delta_F(x-y) = (\not{D}_x + m) \Delta_F(x-y) = -\delta^x(x-y) \\ \not{D}_x S(x-y) = (i\gamma \cdot \partial_x - M) S(x-y) = \delta^x(x-y) \\ S(x-y) \overleftarrow{\not{D}_x} = S(x-y) (-i\overleftarrow{\gamma} \cdot \partial_x - M) = \delta^x(x-y) \end{cases}$$

$$\begin{aligned} S_{fi} &= -2ig \int dx_1 \dots dx_4 dy_1 dy_2 e^{ikx_2} e^{ip'x_4} \bar{u}^{s'}(p') \delta^x(x_2 - y_2) \\ &\quad \times \delta^x(x_4 - y_1) \gamma_5 S(y_1 - y_2) \gamma_5 \delta^x(y_2 - x_3) \delta^x(y_1 - x_1) u^s(p) e^{-ipx_3} e^{-ikx_1} \\ &= -2ig^2 \int dy_1 dy_2 e^{i(p-k)y_1} e^{i(k'-p)y_2} \bar{u}^{s'}(p') \gamma_5 S(y_1 - y_2) \gamma_5 u^s(p) \end{aligned}$$

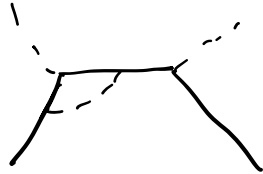
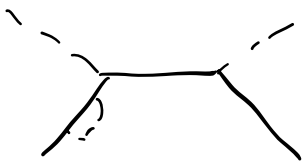
$$S(x) \Delta_F(x) \\ S(y_1 - y_2) = \frac{1}{(2\pi)^4} \int dq \frac{\gamma^\mu q_\mu + M}{q^2 - m^2} e^{-iq(y_1 - y_2)}$$

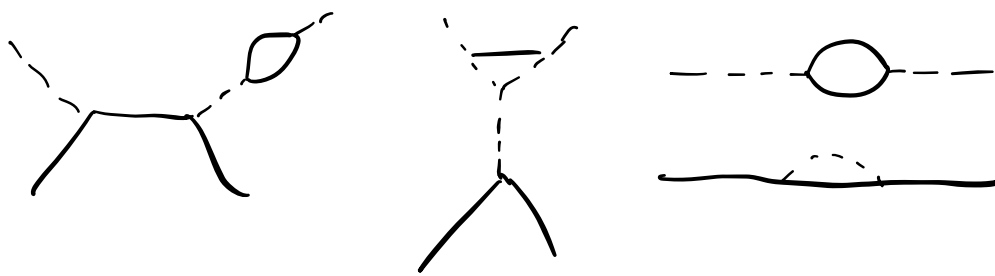
$$\begin{aligned} \int dy_1 dy_2 e^{i(p-k)y_1} e^{i(k'-p)y_2} e^{-iq(y_1 - y_2)} &= (2\pi)^8 \delta^4(p-k-q) \delta^4(k'-p+q) \\ &= (2\pi)^8 \delta^4(q-p+k') \delta^4(p'+k'-p-k) \\ &= (2\pi)^8 \delta^4(q-p+k') \delta^4(p_f - p_i) \end{aligned}$$

$$p+k' \Leftrightarrow p+k$$

$$\begin{aligned} S_{fi} &= -i \delta^4(p_f - p_i) 2g^2 (2\pi)^4 \bar{u}^{s'}(p') \gamma_5 \frac{\gamma(p-k') + M}{(p-k')^2 - m^2} \gamma_5 u^s(p) \\ &= i \delta^4(p_f - p_i) 2g^2 (2\pi)^4 \bar{u}^{s'}(p') \gamma \cdot k' u^s(p) \frac{1}{2p \cdot k' - m^2} \end{aligned}$$

1. nAq → nA72Z





2. 费米子 spinor:  $u(p)$  ( $v(p)$  反粒子), 费米子 spinor:  $\bar{u}(p)$

3. 对于顶角, 耦合 int:  $ig$  费米子耦合 int:  $i\gamma_5 g$  标量  $(2\pi)^4 \delta^4$

4. spinor prop:  $\frac{i}{(2\pi)^4} \frac{i}{(\not{x}p - m)}$   $d^4p$

5. (pseudo) scalar prop:  $\frac{i}{(2\pi)^4} \frac{i}{p^2 - m^2}$   $d^4p$

6. 积分 (分部积分)

Scattering cross-section

Lorentz invariant  $M$

$$\langle p'_1, p'_2, \dots, | S - 1 | p_1, p_2, \dots \rangle$$

$$= (2\pi)^4 \delta^4(p_f - p_i) i M(p_1, p_2, \dots, p'_1, p'_2, \dots)$$

$p_1, p_2$

$$|i\rangle = \int \frac{d^3k_1}{(2\pi)^3 2k_1} \frac{d^3k_2}{(2\pi)^3 2k_2} f(k_1) g(k_2) |k_1, k_2\rangle$$

$$k_1 \approx p_1, k_2 \approx p_2$$

$$|f\rangle = |p'_1, p'_2\rangle$$

$$\int d\tilde{k}_1 d\tilde{k}_2 f(k_1) g(k_2) \langle p_i p_i' | S^{-1} | k_1 k_2 \rangle$$

$$= (2\pi)^4 i \int d\tilde{k}_1 d\tilde{k}_2 f(k_1) g(k_2) \delta(p_i + p_i' - k_1 - k_2) M(p_i', p_i, k_1, k_2)$$

$$W = (2\pi)^8 \int d\tilde{k}_1 d\tilde{k}_2 d\tilde{q}_1 d\tilde{q}_2 f(k_1) g(k_2) f^*(q_1) g^*(q_2)$$

$$\delta(p_i + p_i' - k_1 - k_2) \delta(p_i + p_i' - q_1 - q_2) M(p_i', p_i, k_1, k_2) M^*(p_i', p_i, q_1, q_2)$$

$$\delta(k_1 + k_2 - q_1 - q_2)$$

$$M(p_i', p_i, p_1, p_2)$$

$$\hat{f}(x) = \int d\tilde{q} e^{i\tilde{q}x} f(q)$$

$$d\tilde{q} = \frac{d^4 q}{(2\pi)^3 2q_0}$$

$$|\hat{f}(x)|^2 = \int d\tilde{k}_1 d\tilde{q}_1 e^{i(k_1 - q_1)x} f(k_1) f^*(q_1)$$

$$|\hat{g}(x)|^2 = \int d\tilde{k}_2 d\tilde{q}_2 e^{i(k_2 - q_2)x} g(k_2) g^*(q_2)$$

$$(2\pi)^4 \delta(k_1 + k_2 - q_1 - q_2) = \int d^4 x e^{i(k_1 + k_2 - q_1 - q_2)x}$$

$$W = \int d^4 x |\hat{f}(x)|^2 |\hat{g}(x)|^2 (2\pi)^4 \delta(p_i + p_i' - p_1 - p_2) |M(p_i', p_i, p_1, p_2)|^2$$

$$\frac{dW}{d^4 x} = |\hat{f}(x)|^2 |\hat{g}(x)|^2 (2\pi)^4 \delta(p_i + p_i' - p_1 - p_2) |M(p_i', p_i, p_1, p_2)|^2$$

$$|\hat{f}(x)|^2 2p_1^0 \quad |\hat{g}(x)|^2 2p_2^0$$

$$p_i = m \quad mc \text{ flux} = 2|\vec{p}_i| |\hat{f}(x)|^2$$

$$\text{tar density} = 2m_2 |\hat{g}(x)|^2$$

$$\frac{dW}{dt dV} = (\text{inc flux}) \times (\text{tar density}) \times d\sigma$$

$$d\sigma = (2\pi)^4 \delta^4(p_1' + p_2' - p_1 - p_2) \frac{1}{4m_1 |p_1|} |M|^2$$

$$m_2 |p_1|$$

$$B = [(p_1 \cdot p_2)' - m_1^2 m_2^2]^{1/2} = m_2 |p_1| \quad \text{실용적}$$

$$d\sigma = \frac{(2\pi)^4}{4B} \frac{d^3 p_1'}{(2\pi)^3 2(p_1')_0} \frac{d^3 p_2'}{(2\pi)^3 2(p_2')_0} \delta^4(p_f - p_i) |M|^2$$

$S_1, S_2$  (spin)

$$|M|^2 \rightarrow \frac{1}{(2S_1+1)(2S_2+1)} \sum_{S_1, S_2} |M_{fi}|^2$$

$p_0/m \quad |\tilde{g}(x)|^2$

$$\frac{d^3 p}{(2\pi)^3 2p_0} \quad \left(\frac{m}{p_0}\right) \left[\frac{d^3 p}{(2\pi)^3}\right]$$

$$d\sigma = \frac{1}{(2\pi)^2} \frac{d^3 p_1'}{2E_1'} \frac{d^3 p_2'}{E_2'/M} \cdot \frac{M}{2B} \delta^4(p_f - p_i) \frac{1}{2} \sum_{\text{spin}} |M_{fi}|^2$$

$$= \frac{1}{32\pi^2} \frac{d^3 p_1'}{E_1'} \frac{d^3 p_2'}{E_2'} \frac{M^2}{B} \delta(E_1' + E_2' - E_i) \delta^3(p_1' + p_2' - p_i) \sum_{\text{spin}} |M_{fi}|^2$$

FCMS:  $\vec{p}_1' = -\vec{p}_2' = \vec{p}_f$

$$I = \int \frac{d^3 p_1'}{E_1'} \frac{d^3 p_2'}{E_2'} \delta(E_1' + E_2' - E_i) \delta^3(p_1' + p_2' - p_i)$$

$$= \int \frac{d^3 p_f}{E_1' E_2'} \delta(E_1' + E_2' - E_i) = \int \frac{p_f^2 dp_f d\Omega_f}{E_1' E_2'} \delta(E_1' + E_2' - E_i)$$

$$\delta(f(x)) = \frac{\delta(x-x_0)}{|f'(x_0)|} \quad f(x_0) = 0$$



$$I = \frac{P_f}{E_i} \int d\Omega_f$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{32\pi^2} \frac{M^2 P_f}{B E_i} \sum_{\text{spin}} |M_{fi}|^2$$

$$B = P_f (E_i + E_{i'}) = P_f W, \quad W \text{ 为系统总能量}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{32\pi^2} \left(\frac{M}{W}\right)^2 \sum_{\text{spin}} |M_{fi}|^2$$

$$M_{fi} = 2g^2 \bar{u}^s(p') \gamma k' u^s(p) \frac{1}{2p \cdot k - m^2}$$

$$\sum_{\text{spin}} |\bar{u}' A u|^2 \quad u' = u^{s'}(p') \quad A$$

$$\begin{aligned} (\bar{u}' A u)^* &= u'^T \gamma^{\alpha} A^{\dagger} u^{\dagger} \\ &= u'^T A^{\dagger} \gamma^{0T} u' \\ &= \bar{u}' \bar{A} u' \end{aligned}$$

$$\sum_{\alpha} u_i^{\alpha}(p) \bar{u}_j^{\alpha}(p) = \left(\frac{\gamma \cdot p + M}{2M}\right)_{ij}$$

$$\text{Tr}(\gamma \cdot a)(\gamma \cdot b) = 4a \cdot b$$

$$\begin{aligned} \text{Tr}(\gamma \cdot a)(\gamma \cdot b)(\gamma \cdot c)(\gamma \cdot d) &= -\text{Tr}(\gamma \cdot b)(\gamma \cdot a)(\gamma \cdot c)(\gamma \cdot d) \\ &\quad + 2a \cdot b \text{Tr}(\gamma \cdot c)(\gamma \cdot d) \end{aligned}$$

$$\begin{aligned} \sum_{\text{spin}} |M_{fi}|^2 &= 4g^4 \left(\frac{1}{2p \cdot k' - m^2}\right)^2 \frac{1}{4m^2} \text{Tr}[(\gamma \cdot p' + M)\gamma \cdot k'(\gamma \cdot p + M)\gamma \cdot k] \\ &= \frac{g^4}{m^2} \left(\frac{1}{2p \cdot k' - m^2}\right)^2 4 \left\{ 2(p \cdot k')(p' \cdot k') + m^2 [M^2 - (p \cdot p')] \right\} \end{aligned}$$

$$|\vec{p}| = |\vec{p}'| = |\vec{k}| = |\vec{k}'| = q$$

$$p = ((q^2 + M^2)^{1/2}, \vec{q}), \quad k = ((q^2 + m^2)^{1/2}, -\vec{q})$$

$$p' = ((q^2 + M^2)^{1/2}, \vec{q}'), \quad k' = ((q^2 + m^2)^{1/2}, -\vec{q}')$$

low energy,  $m, M \gg q$   $p \cdot k' \approx Mm$   $p' \cdot k' \approx Mm$   $p \cdot p' \approx M^2$

$$\sum_{\text{spin}} |M_{fi}|^2 \approx \frac{g^4}{(2M-m)^2}$$

$$W \approx M + m$$

$$\frac{d\sigma}{d\Omega} = \frac{g^4}{4\pi^2} \left(\frac{M}{M+m}\right)^2 \frac{1}{(2M-m)^2} \approx \frac{g^4}{16\pi} \frac{1}{M^2}$$

Yukawa potential:

$$V = \frac{g e^{-\eta r}}{r}$$

$$\eta = \alpha^{-1}$$

pion Compton ~~length~~

$$1.4 \times 10^{-15} \text{ m}$$

$$r = 2.8 \times 10^{-15} \text{ m}$$

$$20 \text{ MeV}$$

$$0.5 \text{ MeV}$$

$$\frac{g^2}{e^2} \approx 40$$

$$\frac{g^2}{\hbar c} \approx 40 \times \frac{e^2}{\hbar c} \approx 0.3$$

$$\alpha = 4\pi \left(\frac{d\sigma}{d\Omega}\right)$$

$$= 4\pi \left(\frac{g^2}{4\pi}\right)^2 \frac{1}{M^2}$$

$$= \frac{1}{4\pi} \left(\frac{g^2}{\hbar c}\right)^2 \left(\frac{\hbar}{Mc}\right)^2$$

$$\approx 120 \mu\text{b}$$

$$1 \text{ b} = 10^{-28} \text{ m}^2$$

$$\mathcal{L}_{int} = \frac{f}{m} \bar{\psi} \gamma_5 \gamma^\mu \bar{c} \psi - \partial_\mu \bar{c} \partial^\mu c$$

$$\frac{f^2}{4\pi} = 0.08$$

f 耦合值

Schweber, Bethe & de Hoffmann

$$g = \left(\frac{2M}{r}\right) + \quad (\text{M 杨度号})$$

$$\frac{g^2}{4a} \approx 15$$

$$\sigma = \underline{4\pi \frac{g^2}{2a} = 486}$$