Path-integral quantisation: gauge fields

Propogators and gauge conditions in QED

Photon propagator — canonical formalism

$$Z[J] = \int \mathcal{D}A_\mu \exp i \int (\mathcal{L} + J^\mu A_\mu) dx$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\iint \mathcal{D}\phi \mathcal{D}\pi \exp i \int (\dot\phi\pi - \mathcal{H}) dx$$

Fadeev - Popov technique

$$\partial_\mu F^{\mu\nu} = 0$$

$$\mathcal{L} = \frac{1}{2} A^\mu [g_{\mu\nu} \Box - \partial_\mu \partial_\nu] A^\nu$$

photon prop: $D_{\mu\nu}$

$$\underline{(g_{\mu\nu} \Box - \partial_\mu \partial_\nu)} D^{\nu\lambda}(x-y) = \delta_\mu^\lambda \delta^4(x-y)$$

$$\partial^\mu D_{\mu\nu} = 0$$

Photon prop — path-integral method

$$Z = \int \mathcal{D} A_\mu e^{i\int \mathcal{L} dt}$$

Gauge fixing term

Lorentz gauge condition: $\partial^\mu A_\mu = 0$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \underline{\frac{1}{2}(\partial_\mu A^\mu)^2} = \mathcal{L}_0 + \underline{\mathcal{L}_{GF}}$$

$$g_{\mu\nu}\Box - \partial_\mu\partial_\nu \rightarrow -g_{\mu\nu}k^2 + k_\mu k_\nu$$

$$(A g^{\nu\lambda} + B k^\nu k^\lambda)(-g_{\mu\nu}k^2 + k_\mu k_\nu) = \delta_\mu^\lambda$$

$$-Ak^2\delta_\mu^\lambda + A k_\mu k^\lambda = \delta_\mu^\lambda$$

$-g_{\mu\nu}k^2$ 有逆 $-g^{\nu\lambda}\left(\frac{1}{k}\right)^2$

$$D_F(k)_{\mu\nu} = -\frac{g_{\mu\nu}}{k}$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 = \frac{1}{2}A^\mu\left[g_{\mu\nu}\Box + \left(\frac{1}{\alpha}-1\right)\partial_\mu\partial_\nu\right]A^\nu$$

α为有限值

$$-k^2 g_{\mu\nu} + \left(1-\frac{1}{\alpha}\right)k_\mu k_\nu$$

$$D(k)_{\mu\nu} = -\frac{1}{k^2}\left[g_{\mu\nu} + (\alpha-1)\frac{k_\mu k_\nu}{k^2}\right]$$

$\alpha \to 1$ 费曼传播子(费曼规范)

$\alpha \to 0$ 朗道规范

Propagator for transverse photons

$k_\mu \varepsilon^\mu = 0$

Columb gauge $\nabla \cdot \vec{A} = 0, \phi = 0$

$$\langle 0| T(A_\mu(x) A_\nu(y)) |0\rangle = i D_{\mu\nu}(x-y)$$

$$\langle 0| T(A_\mu(x) A_\nu(y)) |0\rangle = \langle 0| \int \frac{d^3k}{(2\pi)^3 2k_0} \frac{d^3k'}{(2\pi)^3 2k_0'} \sum_{\lambda\lambda'=1}^{2} \varepsilon_\mu^{(\lambda)}(k) \varepsilon_\nu^{(\lambda')}(k')$$
$$\times \{[a^{(\lambda)}(k) e^{-ikx} + a^{(\lambda)\dagger}(k) e^{ikx}]$$
$$\times [a^{(\lambda')}(k') e^{-ik'y} + a^{(\lambda')\dagger}(k') e^{ik'y}] \theta(x_0-y_0)$$
$$+ [a^{(\lambda')}(k') e^{-ik'y} + a^{(\lambda')\dagger}(k') e^{ik'y}]$$
$$\times [a^{(\lambda)}(k) e^{-ikx} + a^{(\lambda)\dagger}(k) e^{ikx}] \theta(y_0-x_0) |0\rangle$$
$$= \langle 0| \int\int \frac{d^3k\, d^3k'}{(2\pi)^6 2k_0 2k_0'} \sum_{\lambda\lambda'=1}^{2} \varepsilon_\mu^{(\lambda)}(k) \varepsilon_\nu^{(\lambda')}(k')$$
$$\times [a^{(\lambda)}(k) a^{(\lambda')\dagger}(k') e^{i(k'y-kx)} \theta(x_0-y_0)$$
$$+ a^{(\lambda')}(k') a^{(\lambda)\dagger}(k) e^{i(kx-k'y)} \theta(y_0-x_0) |0\rangle$$

$$\langle 0| T(A_\mu(x) A_\mu(y)) |0\rangle = \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=1}^{2} \varepsilon_\mu^{(\lambda)}(k) \varepsilon_\nu^{(\lambda)}(k)$$
$$\times [e^{ik(y-x)} \theta(x_0-y_0) + e^{ik(x-y)} \theta(y_0-x_0)]$$

$$\Delta_F(x, m=0) = -i \int \frac{d^3k}{(2\pi)^3 2k_0} [\theta(x_0) e^{-ikx} + \theta(-x_0) e^{ikx}]$$
$$= \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{k^2 + i\varepsilon}$$

$$\langle 0| T(A_\mu(x) A_\mu(y)) |0\rangle = i \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik(x-y)}}{k^2 + i\varepsilon} \sum_{\lambda=1}^{2} \varepsilon_\mu^{(\lambda)}(k) \varepsilon_\nu^{(\lambda)}(k)$$

$$D_{\mu\nu}^{tr}(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik(x-y)}}{k^2 + i\varepsilon} \underline{\sum_{\lambda=1}^{2} \varepsilon_\mu^{(\lambda)}(k) \varepsilon_\nu^{(\lambda)}(k)}$$

$\eta_\mu = (1, 0, 0, 0)$ $\qquad \varepsilon_\mu^{(1,2)}, \eta_\mu \quad k_\mu$

$$\bar{k}_\mu = \frac{k^\mu - (k\cdot\eta)\eta^\mu}{[(k\cdot\eta)^2 - k^2]^{1/2}}$$

$k \cdot \varepsilon = 0, \quad \eta \cdot \varepsilon = 0$

$$g_{\mu\nu} = \eta_\mu \eta_\nu - \sum_{\lambda=1}^{2} \varepsilon_\mu^{(\lambda)}(k)\, \varepsilon_\nu^{(\lambda)}(k) - \bar{k}_\mu \bar{k}_\nu$$

$$\begin{aligned}\sum_{\lambda=1}^{2} \varepsilon_\mu^{(\lambda)}(k)\, \varepsilon_\nu^{(\lambda)}(k) &= \eta_\mu \eta_\nu - g_{\mu\nu} - \bar{k}_\mu \bar{k}_\nu \\ &= -g_{\mu\nu} - \frac{k_\mu k_\nu}{(k\cdot\eta)^2 - k^2} + \frac{(k\cdot\eta)(k_\mu \eta_\nu + \eta_\mu k_\nu)}{(k\cdot\eta)^2 - k^2} - \frac{k^2 \eta_\mu \eta_\nu}{(k\cdot\eta)^2 - k^2}\end{aligned}$$